The Key Player Problem in High-Voltage Power Networks

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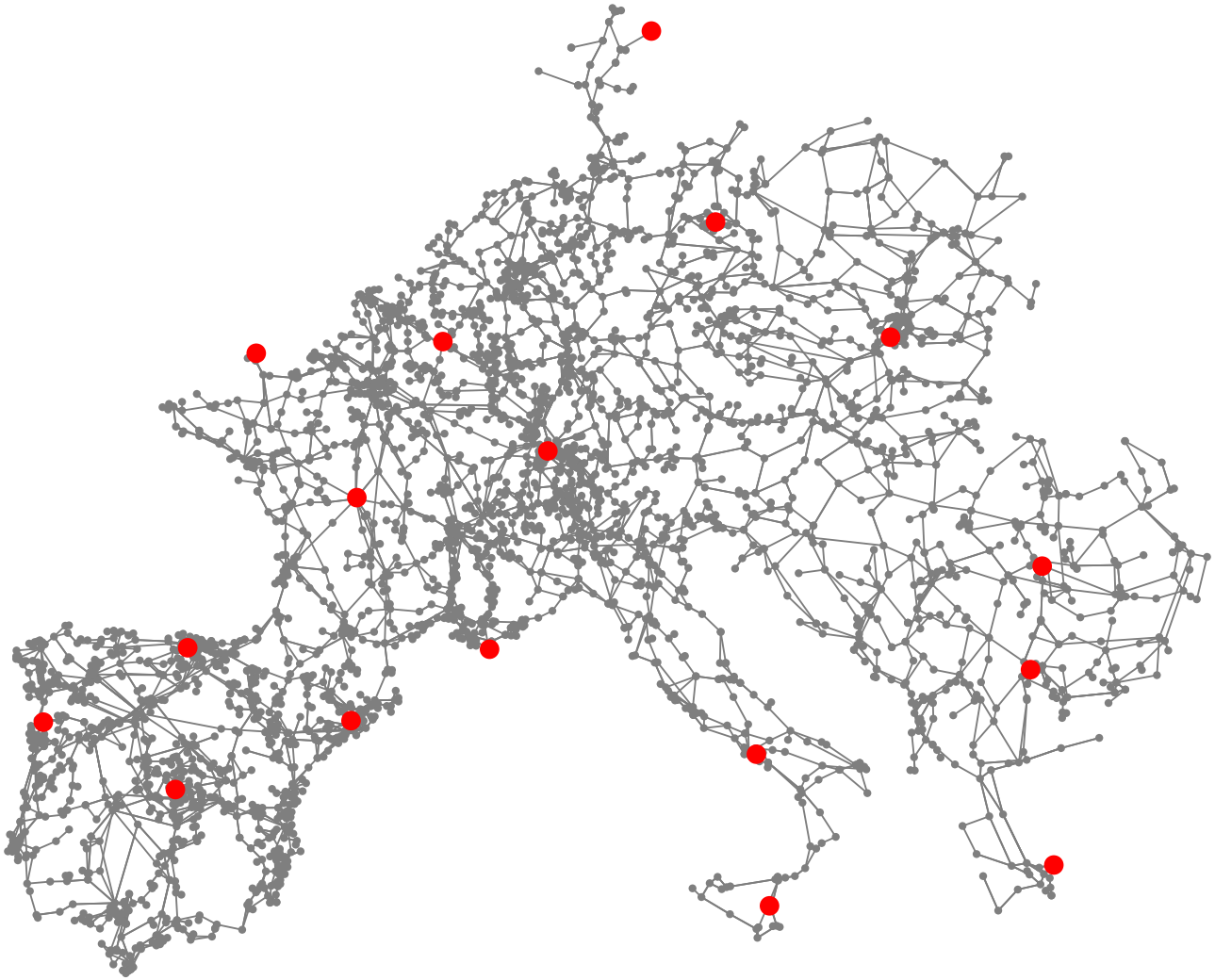
Satellite Conference on Complex Systems 2020, Complexity in Energy Systems.

December 9, 2020, etranselec.ch

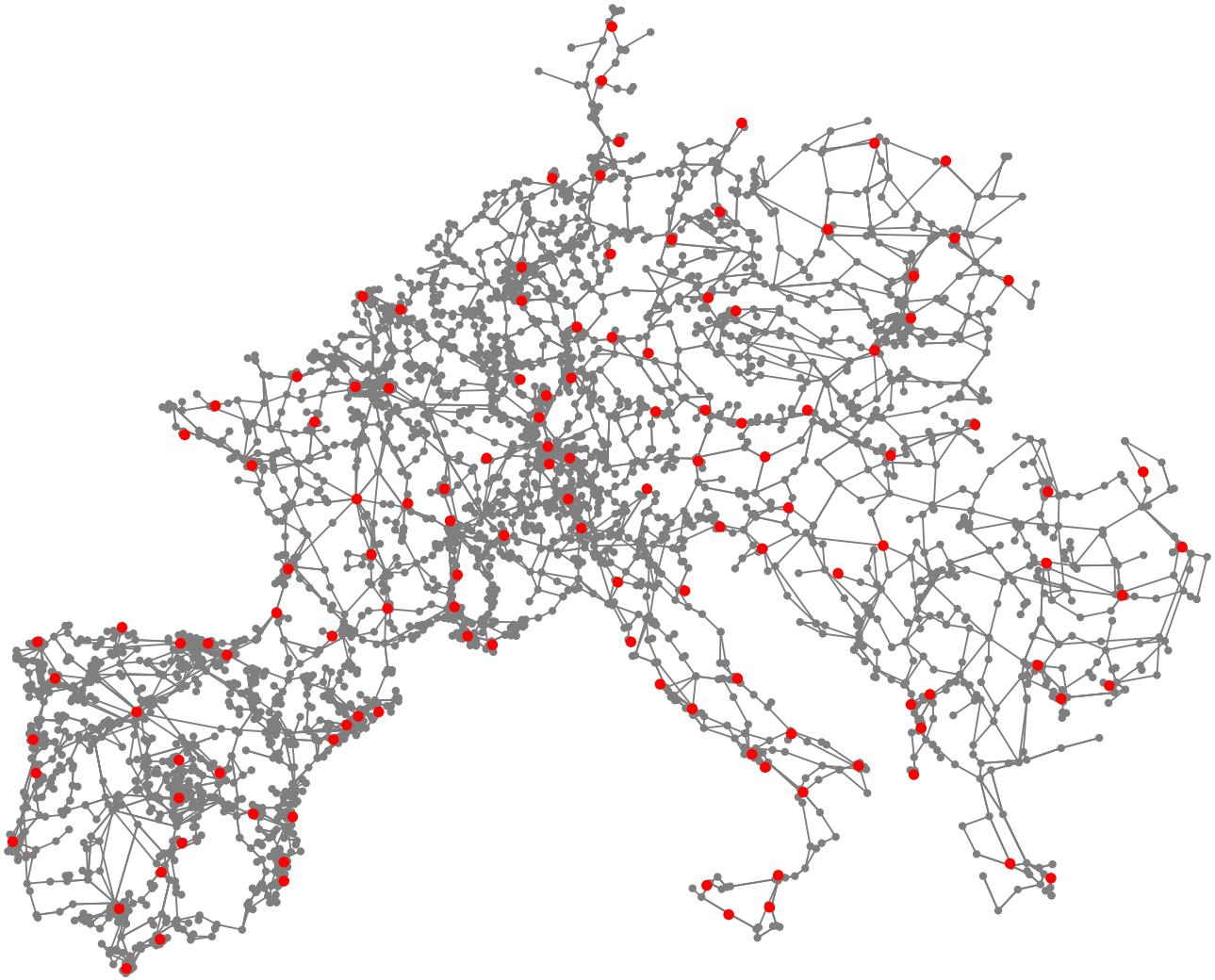
MT, L. Pagnier, P. Jacquod *Science Advances* **5**(11), eaaw8359 (2019)

The Key Player Problem in High-Voltage Power Networks



Centralized power generation

The Key Player Problem in High-Voltage Power Networks



Decentralized power generation

Coupled phase dynamics

Swing Equations in the lossless line limit (second-order Kuramoto):

$$m_i\ddot{\theta}_i + d_i\dot{\theta}_i = P_i - \sum_j b_{ij}\sin(\theta_i - \theta_j) \ , \ i = 1, ..., n.$$

$b_{ij} = b_{ji} \geq 0$.
P_i: injected/consumed power.
m_i: inertia (only at generators in realistic setting).
d_i: damping from control.

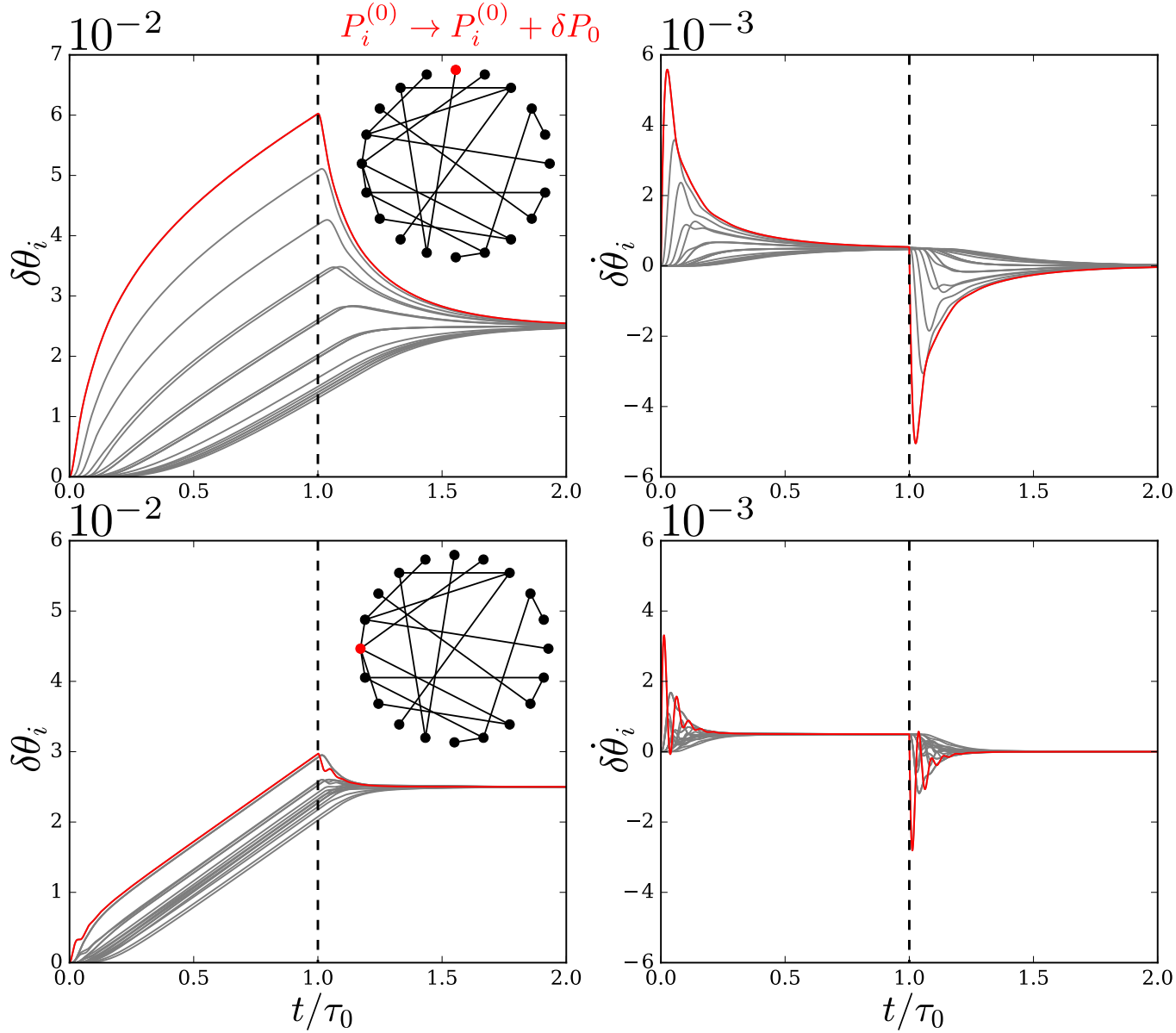
Steady-state solutions: Synchronous state $\{\theta_i^{(0)}\}$ such that:

$$P_i = \sum_j b_{ij}\sin(\theta_i^{(0)} - \theta_j^{(0)}) \ , \ i = 1, ..., n.$$

$\sum_i P_i = 0$.

Perturbations: $P_i \to P_i^{(0)} + \delta P_i(t)$.

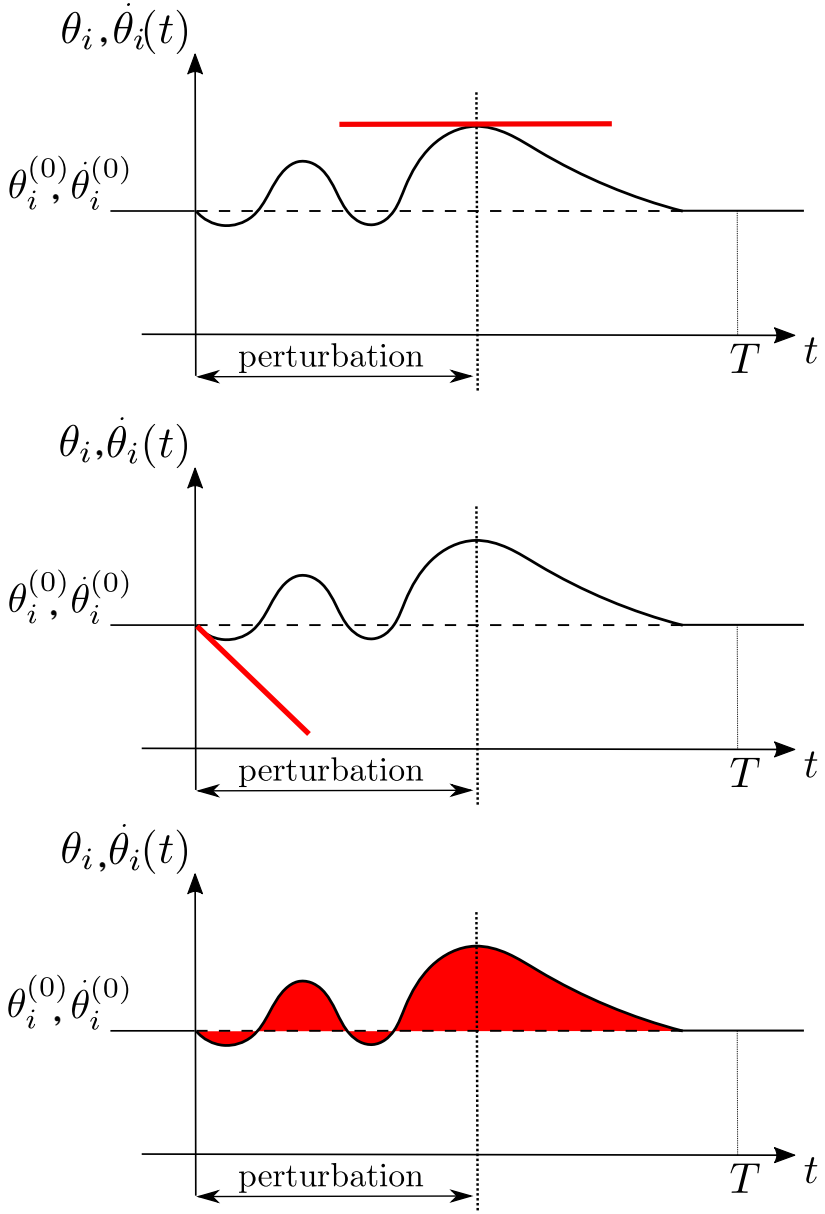
Coupled phase dynamics: Transient excursion



MT and Jacquod *Phys. Rev. E* **100**(3), 032303 (2019).

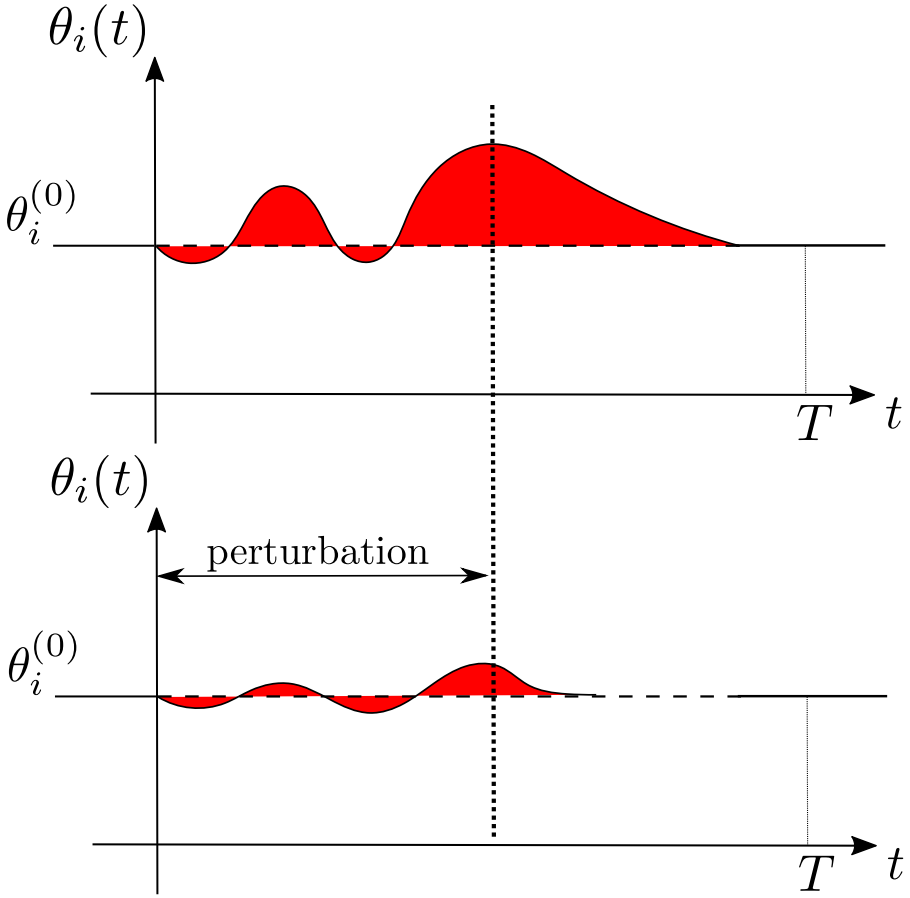
Quantifying Robustness

- Maximum of the response, $\max_t(\theta_i)$.
- Rate of change of frequency (RoCoF), $\ddot{\theta}_i$.
- Performance measure (quadratic integrals over the transient).



Quantifying Robustness

Performance measures :



$$\mathcal{P}_1(T) = \sum_i \int_0^T |\theta_i(t) - \theta_i{}^{(0)}|^2 \mathrm{d}t \,,$$

$$\mathcal{P}_2(T) = \sum_i \int_0^T |\dot{\theta}_i(t) - \dot{\theta}_i^{(0)}|^2 \mathrm{d}t \,.$$

$$\mathcal{P}_{1,2}^{\infty} = \mathcal{P}_{1,2}(T \to \infty) \,.$$

Noisy disturbances $\to$ divide by T.

Perturbations : $P_i \to P_i^{(0)} + \delta P_i(t)$.

Response to Perturbations: Linearization

Linear response: Perturbation of the natural frequencies.

$P_i(t) = P_i^{(0)} + \delta P_i(t) \rightarrow \theta_i(t) = \theta_i^{(0)} + \delta\theta_i(t)$:

$$m\,\delta\ddot{\boldsymbol{\theta}}(t) + d\,\delta\dot{\boldsymbol{\theta}}(t) = \delta P(t) - \mathbb{L}(\{\theta_i^{(0)}\})\delta\boldsymbol{\theta}(t) \ ,$$

Assumption: Homogeneous inertia and damping coefficients.

Response to Perturbations: Linearization

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Assumption: Homogeneous inertia and damping coefficients.

Realistic power grids: inertia on some generators and inertialess loads.

$$\gamma^{-1}\,\delta\ddot{\boldsymbol{\varphi}}(t) + \delta\dot{\boldsymbol{\varphi}}(t) = D^{-1/2}\delta P(t) - D^{-1/2}\mathbb{L}(\{\theta_i^{(0)}\})D^{-1/2}\delta\boldsymbol{\varphi}(t)\ ,$$

where $\delta\boldsymbol{\varphi}(t) = D^{1/2}\delta\boldsymbol{\theta}(t)$.
$\to$ MT, Jacquod *IEEE Control Systems Letters* **5** (3), 929-934 (2021)
presentation next week at CDC2020 December 15.

Response to Perturbations: Linearization

Linear response: Perturbation of the natural frequencies.

- $P_i(t) = P_i^{(0)} + \delta P_i(t) \rightarrow \theta_i(t) = \theta_i^{(0)} + \delta\theta_i(t)$:

$$m\delta\ddot{\boldsymbol{\theta}}(t) + d\delta\dot{\boldsymbol{\theta}}(t) = \delta \mathrm{P}(t) - \mathbb{L}(\{\theta_i^{(0)}\})\delta\boldsymbol{\theta}(t) \ ,$$

$\mathbb{L}(\{\theta_i^{(0)}\})$: the weighted Laplacian matrix,

$$\mathbb{L}_{ij}(\{\theta_i^{(0)}\}) = \begin{cases} -b_{ij}\cos(\theta_i^{(0)} - \theta_j^{(0)}) \, , & i \neq j \, , \\ \sum_k b_{ik}\cos(\theta_i^{(0)} - \theta_k^{(0)}) \, , & i = j \, . \end{cases}$$

Topology $\rightarrow b_{ij}$.

Steady state $\rightarrow \{\theta_i^{(0)}\}$.

Response to Perturbations: Linearization

Linear response: Perturbation of the natural frequencies.

- $P_i(t) = P_i^{(0)} + \delta P_i(t) \rightarrow \theta_i(t) = \theta_i^{(0)} + \delta\theta_i(t)$:

$$m\delta\ddot{\boldsymbol{\theta}}(t) + d\delta\dot{\boldsymbol{\theta}}(t) = \delta \mathrm{P}(t) - \mathbb{L}(\{\theta_i^{(0)}\})\delta\boldsymbol{\theta}(t) \ .$$

Expanding on the eigenvectors u_α of $\mathbb{L}$, we have $\delta\boldsymbol{\theta}(t) = \sum_\alpha c_\alpha(t)\mathrm{u}_\alpha$.

$$c_\alpha(t) = m^{-1}\, e^{-(\gamma+\Gamma_\alpha)t/2} \int_0^t e^{\Gamma_\alpha t_1} \int_0^{t_1} \delta \mathrm{P}(t_2)\cdot \mathrm{u}_\alpha\, e^{(\gamma-\Gamma_\alpha)t_2/2}\, \mathrm{d}t_2 \mathrm{d}t_1 \ ,$$

$\gamma = d/m$ and $\Gamma_\alpha = \sqrt{\gamma^2 - 4\lambda_\alpha/m}$.

MT and Jacquod *Phys. Rev. E* **100**(3), 032303 (2019).
MT, L. Pagnier, P. Jacquod *Science Advances* **5**(11), eaaw8359 (2019)

Response to Perturbations: Time Scales

Intrinsic Time Scales

- Individual elements: m/d.
- Network relaxation: d/λ_α with $\{\lambda_\alpha\}$ the eigenvalues of $\mathbb{L}$.

Perturbation Time Scale

- Correlation time of the external perturbation $\delta \mathrm{P}(t)$.

Quench perturbations

- $\delta P_i(t) = \delta P_{0i} \Theta(t) \Theta(\tau_0 - t)$.

Quench duration $\rightarrow \tau_0$.

Noisy time correlated perturbations

- $\langle \delta P_i(t) P_j(t') \rangle = \delta P_{0i}^2 \delta_{ij} \exp[-|t - t'|/\tau_0]$.

Correlation time $\rightarrow \tau_0$.

Performance Measures

Performance Measures for Noisy Perturbations

$$\mathcal{P}_1^{\infty} = \delta P_0^2 \sum_{\alpha \geq 2} \frac{u_{\alpha,k}^2 (\tau_0 + m/d)}{\lambda_\alpha (\lambda_\alpha \tau_0 + d + m/\tau_0)} \quad , \tag{1}$$

$$\mathcal{P}_2^{\infty} = \delta P_0^2 \sum_{\alpha \geq 2} \frac{u_{\alpha,k}^2}{d(\lambda_\alpha \tau_0 + d + m/\tau_0)} \quad . \tag{2}$$

Performance Measures Asymptotics

Short time correlation: $\tau_0 \ll d/\lambda_\alpha, m/d$

$$\mathcal{P}_1^\infty \simeq \delta P_0^2 \frac{\tau_0}{d} \sum_{\alpha \geq 2} \frac{u_{\alpha,k}^2}{\lambda_\alpha} , \tag{3}$$

$$\mathcal{P}_2^\infty \simeq \delta P_0^2 \frac{\tau_0}{dm} \sum_{\alpha \geq 2} u_{\alpha,k}^2 . \tag{4}$$

Long time correlation: $\tau_0 \gg d/\lambda_\alpha, m/d$

$$\mathcal{P}_1^\infty \simeq \delta P_0^2 \sum_{\alpha \geq 2} \frac{u_{\alpha,k}^2}{\lambda_\alpha^2} , \tag{5}$$

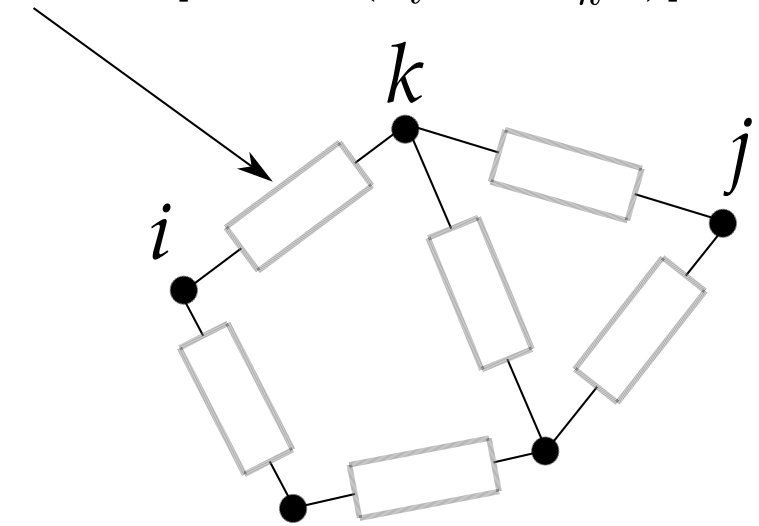
$$\mathcal{P}_2^\infty \simeq \delta P_0^2 \frac{1}{d\tau_0} \sum_{\alpha \geq 2} \frac{u_{\alpha,k}^2}{\lambda_\alpha} . \tag{6}$$

Resistance Distance

$$\Omega_{ij} \;=\; \mathbb{L}^{\dagger}_{ii} + \mathbb{L}^{\dagger}_{jj} - \mathbb{L}^{\dagger}_{ij} - \mathbb{L}^{\dagger}_{ji} = \sum_{\alpha \geq 2} \frac{(u_{\alpha,i} - u_{\alpha,j})^2}{\lambda_\alpha} \; .$$

$\mathbb{L}^{\dagger}$: pseudo inverse of $\mathbb{L}$ (because of $\lambda_1 = 0$).

$$R_{ik} = [b_{ik} \cos(\theta_i^{(0)} - \theta_k^{(0)})]^{-1}$$



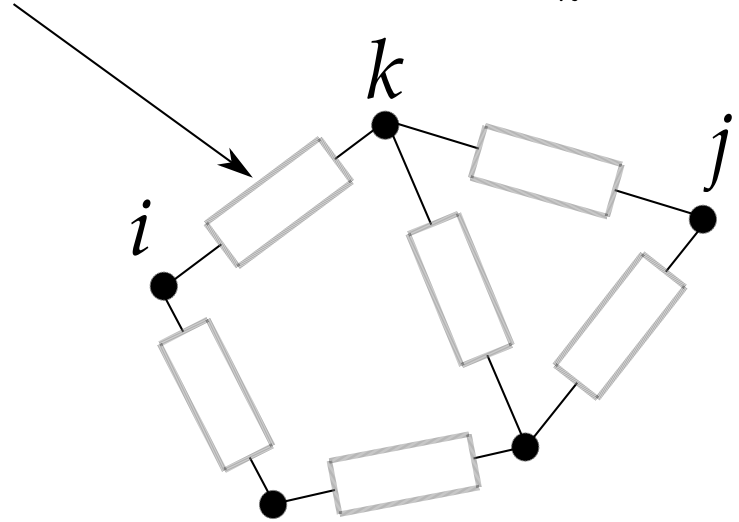
Klein and Randić, *J. Math. Chem.* **12**, 81 (1993).

Resistance Distances, $Kf_p's$ and C_p's

Kirchhoff Index

$$Kf_1 = \sum_{i<j} \Omega_{ij} = n \sum_{\alpha \geq 2} \lambda_\alpha^{-1} \ .$$

$$R_{ik} = [b_{ik} \cos(\theta_i^{(0)} - \theta_k^{(0)})]^{-1}$$



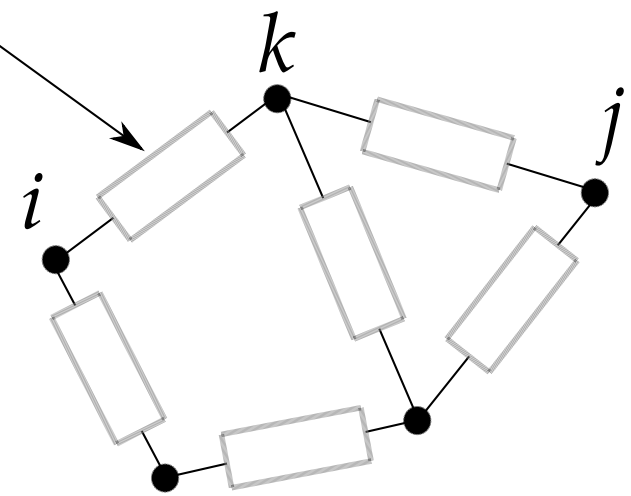
Klein and Randić, *J. Math. Chem.* **12**, 81 (1993).

Resistance Distances, $Kf_p's$ and C_p's

Resistance Centrality

$$C_1(k) = \left[n^{-1} \sum_j \Omega_{kj} \right]^{-1} = \left[\sum_{\alpha \geq 2} \frac{u_{\alpha,k}^2}{\lambda_\alpha} + n^{-2} Kf_1 \right]^{-1}.$$

$$R_{ik} = [b_{ik} \cos(\theta_i^{(0)} - \theta_k^{(0)})]^{-1}$$



Resistance Distances, $Kf'_p s$ and C_p's

Generalized Resistance Distances

$$\Omega_{ij}^{(p)} = \mathbb{L}'^{\dagger}_{ii} + \mathbb{L}'^{\dagger}_{jj} - \mathbb{L}'^{\dagger}_{ij} - \mathbb{L}'^{\dagger}_{ji} = \sum_{\alpha \geq 2} \frac{(u_{\alpha,i} - u_{\alpha,j})^2}{\lambda_\alpha^p} \ ,$$

$$\mathbb{L}' = \mathbb{L}^p \ .$$

Generalized Kirchhoff Indices

$$Kf_p = \sum_{i<j} \Omega_{ij}^{(p)} = n \sum_{\alpha \geq 2} \lambda_\alpha^{-p} \ .$$

Generalized Resistance Centralities

$$C_p(k) = \left[n^{-1} \sum_j \Omega_{kj}^{(p)} \right]^{-1} = \left[\sum_{\alpha \geq 2} \frac{u_{\alpha,k}^2}{\lambda_\alpha^p} + n^{-2} Kf_p \right]^{-1} \ .$$

Specific Local Vulnerabilities $\rightarrow C_p$'s

Local Vulnerability

Perturbing a specific node k i.e. $\delta P_{0i} = \delta_{ik}\delta P_0$,

$\tau_0 \ll m/d$, $(\gamma \pm \Gamma_\alpha)^{-1}$

$$\begin{aligned}\mathcal{P}_1^\infty(k) &= \frac{\delta P_0^2 \tau_0^2}{2d}[C_1^{-1}(k) - n^{-2}Kf_1]\,,\\ \mathcal{P}_2^\infty(k) &= \frac{\delta P_0^2 \tau_0^2}{2md}\frac{(n-1)}{n}\,,\end{aligned}$$

$\tau_0 \gg m/d$, $(\gamma \pm \Gamma_\alpha)^{-1}$

$$\begin{aligned}\mathcal{P}_1^\infty(k) &= \delta P_0^2 \tau_0[C_2^{-1}(k) - n^{-2}Kf_2]\,,\\ \mathcal{P}_2^\infty(k) &= \frac{\delta P_0^2}{d}[C_1^{-1}(k) - n^{-2}Kf_1]\,.\end{aligned}$$

MT, Coletta and Jacquod, *Phys. Rev. Lett.* **120**, 084101 (2018).

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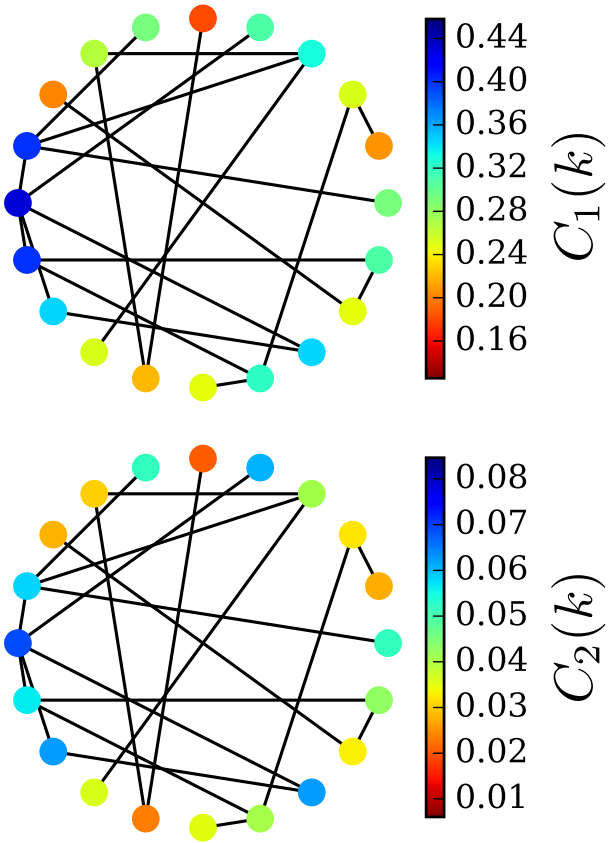
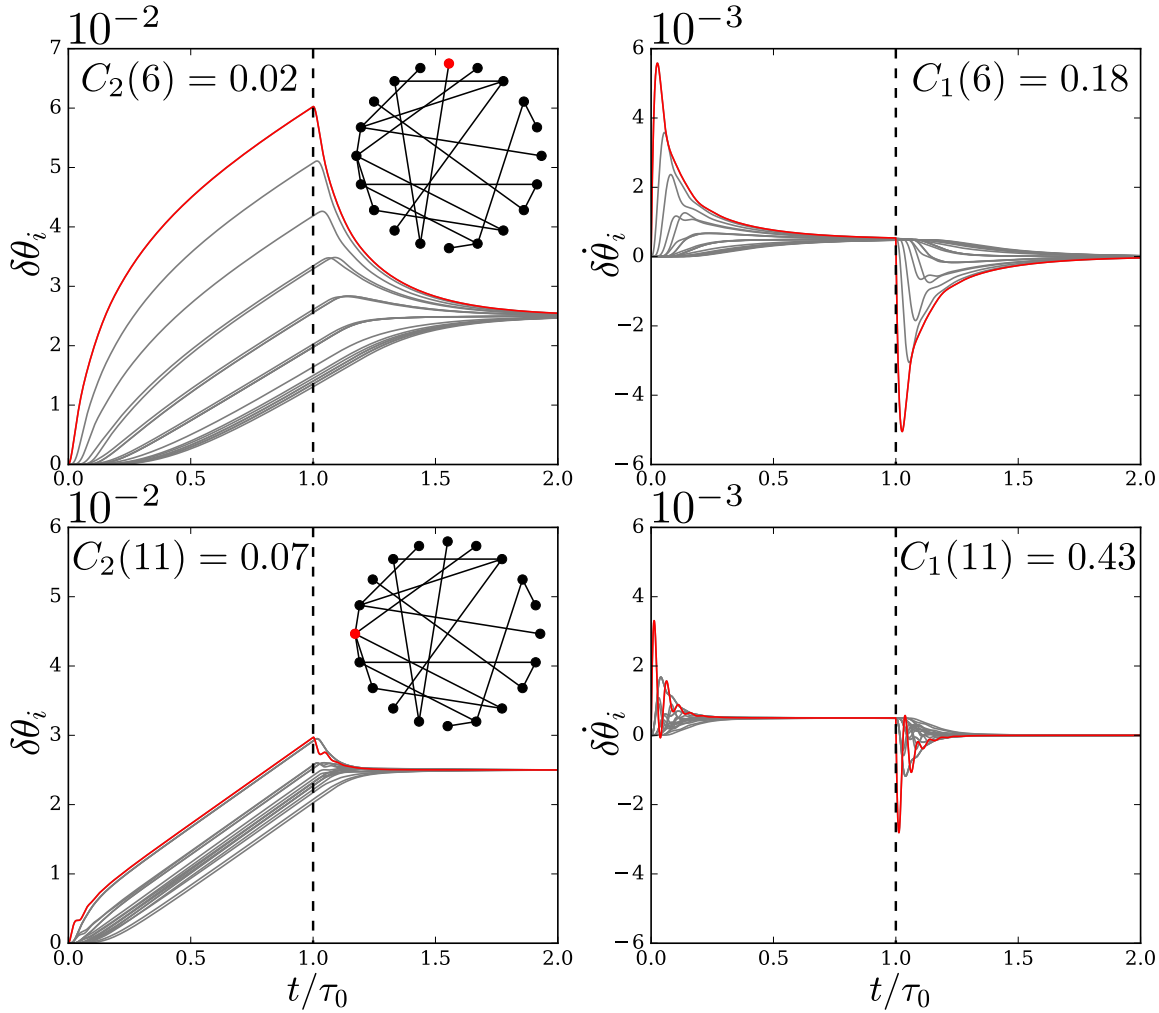
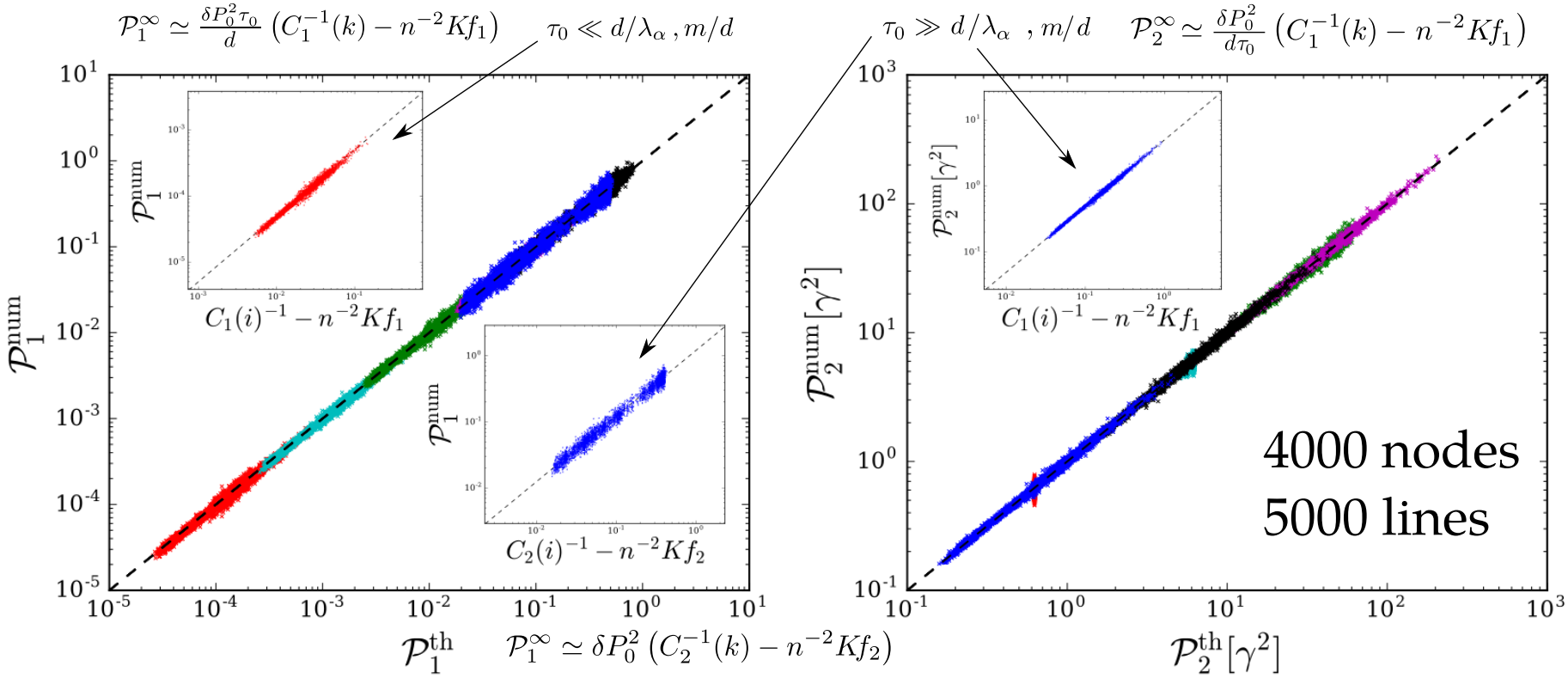
Specific Local Vulnerabilities and C_p's: Numerics





MT and Jacquod *Phys. Rev. E* **100**(3), 032303 (2019).

Specific Local Vulnerabilities and C_p's: Numerics



Physical Realization : European Electrical Grid

$\tau_0 \ll d/\lambda_\alpha\,, m/d$

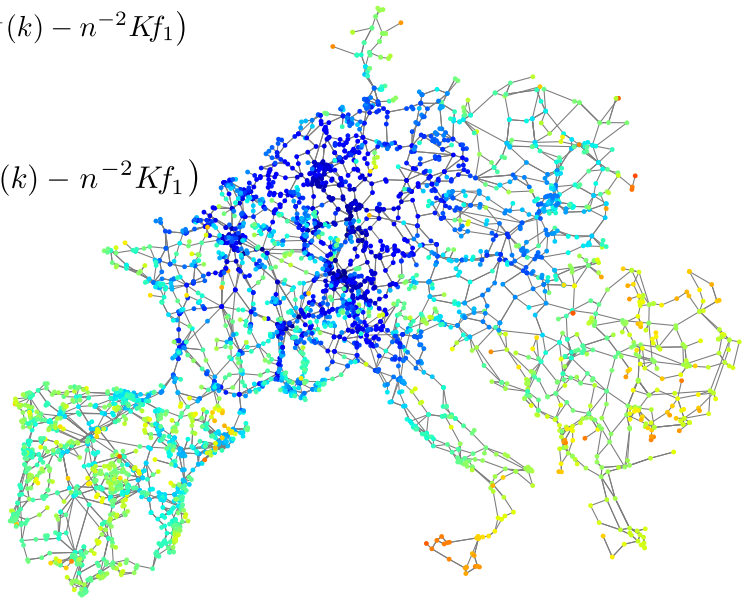
$\mathcal{P}_1^\infty \simeq \frac{\delta P_0^2 \tau_0}{d}\left(C_1^{-1}(k) - n^{-2} K f_1\right)$

$\mathcal{P}_2^\infty \simeq \frac{\delta P_0^2 \tau_0}{dm}\frac{(n-1)}{n}$

$\tau_0 \gg d/\lambda_\alpha\ \ , m/d$

$\mathcal{P}_2^\infty \simeq \frac{\delta P_0^2}{d\tau_0}\left(C_1^{-1}(k) - n^{-2} K f_1\right)$

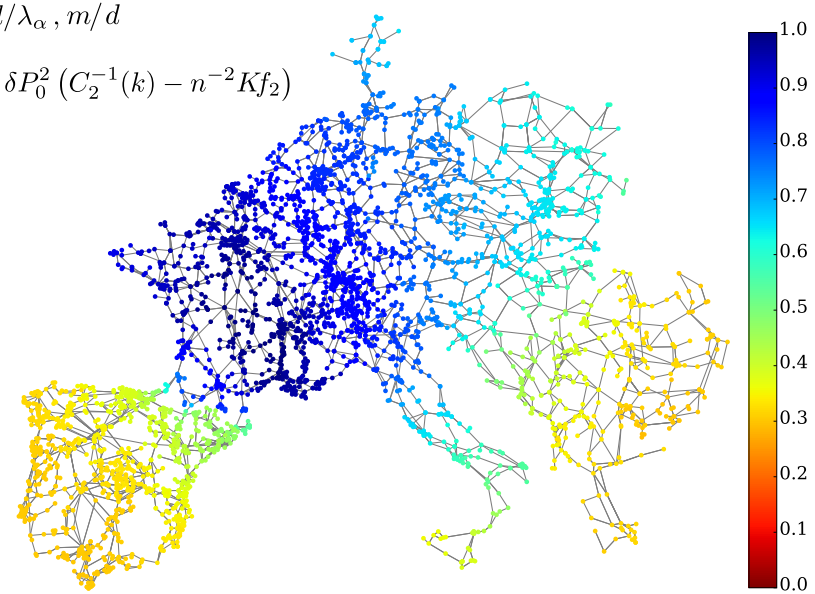
$C_1(i)/\max[C_1(i)]$



$C_2(i)/\max[C_2(i)]$

$\tau_0 \gg d/\lambda_\alpha\,, m/d$

$\mathcal{P}_1^\infty \simeq \delta P_0^2\left(C_2^{-1}(k) - n^{-2} K f_2\right)$



Rankings of Vulnerabilities

Ranking of the nodes

$\rightarrow \mathrm{WLRank}_{1,2}$: Based on $C_{1,2}$ related to

$$\mathbb{L}_{ij}(\{\theta_i^{(0)}\}) = \begin{cases} -b_{ij}\cos(\theta_i^{(0)} - \theta_j^{(0)}), & i \neq j, \\ \sum_k b_{ik}\cos(\theta_i^{(0)} - \theta_k^{(0)}), & i = j. \end{cases}$$

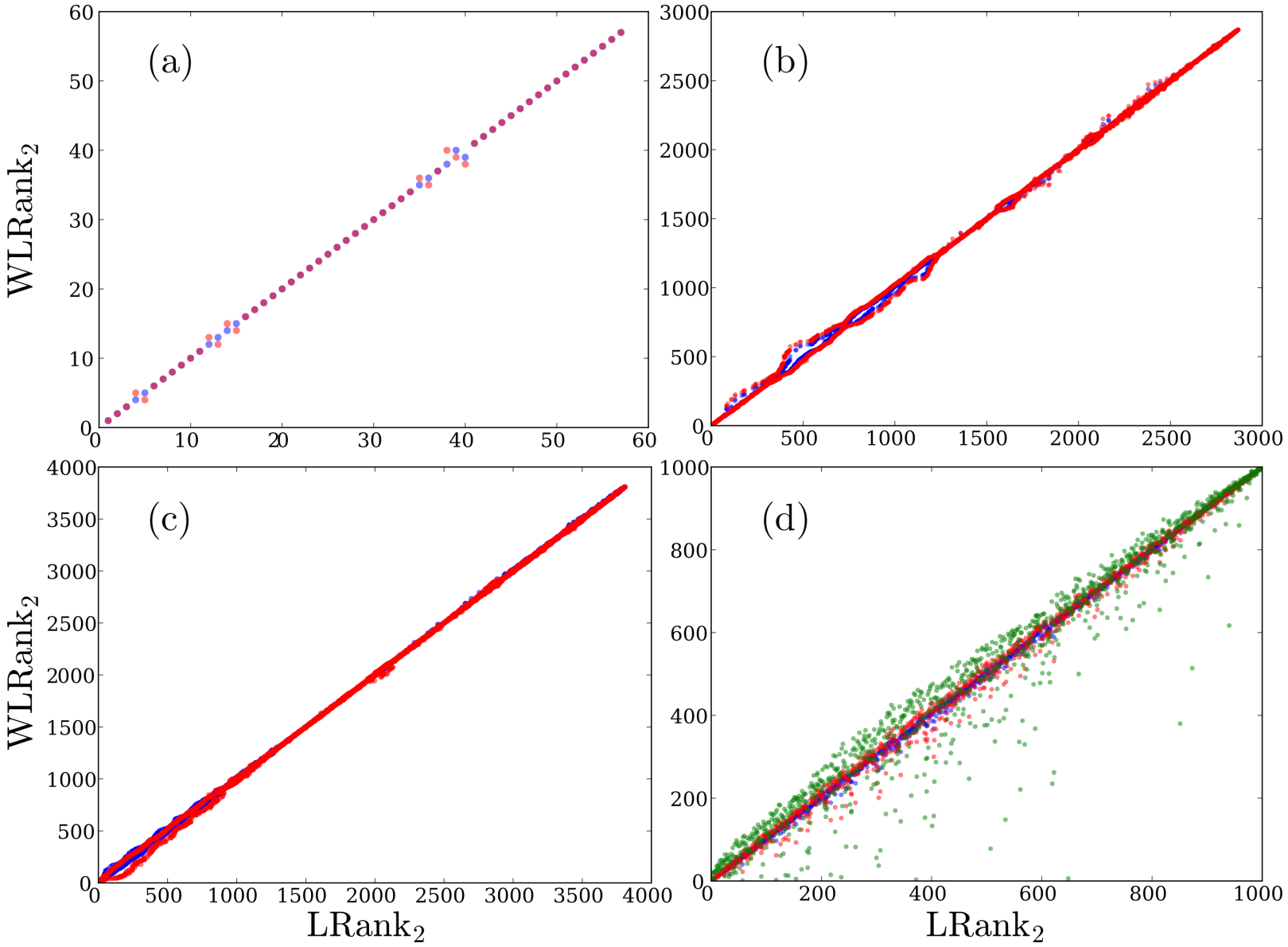
Depend on the operational state.

$\rightarrow \mathrm{LRank}_{1,2}$: Based on $C_{1,2}$ related to

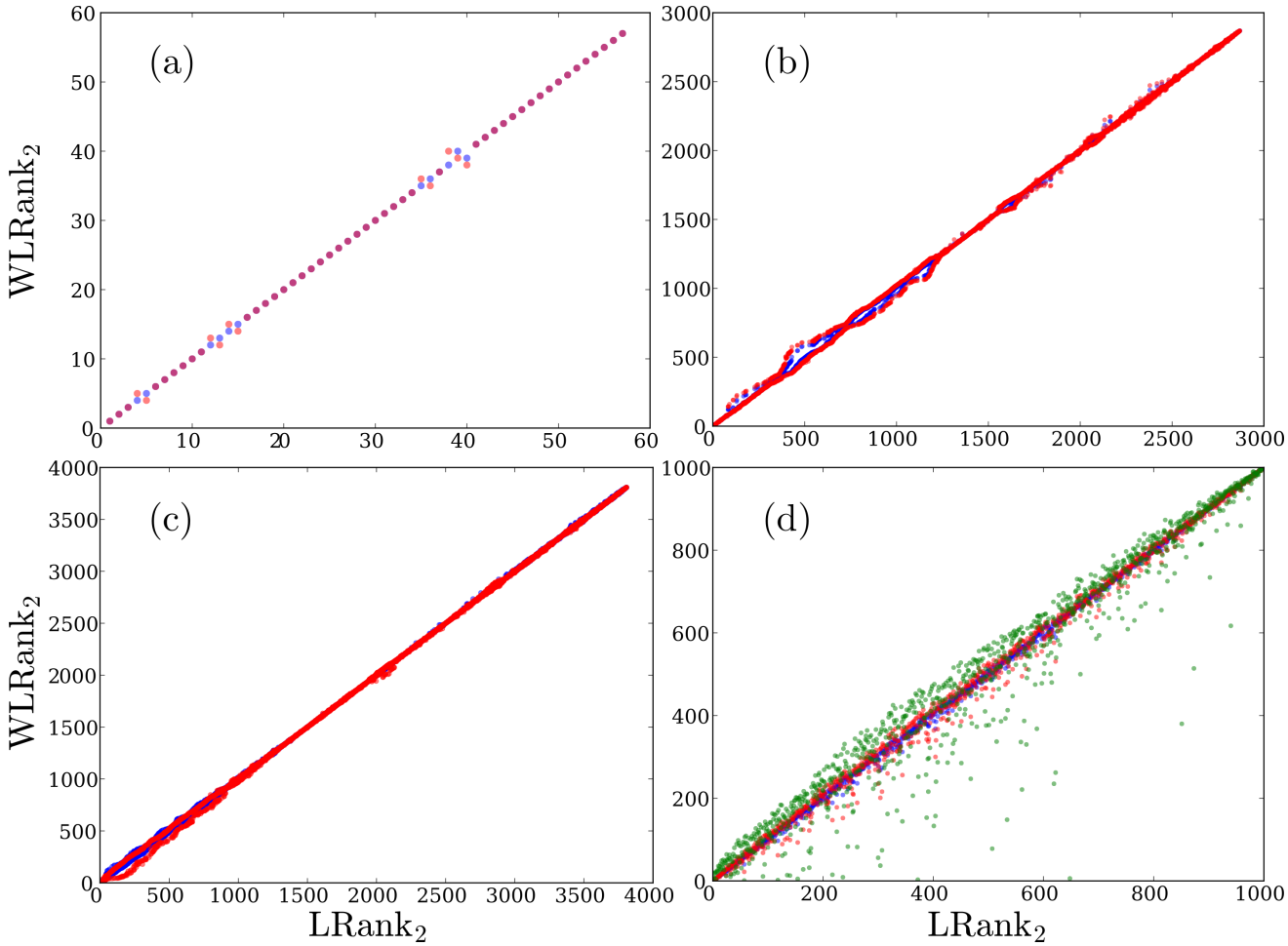
$$\mathbb{L}_{ij} = \begin{cases} -b_{ij}, & i \neq j, \\ \sum_k b_{ik}, & i = j. \end{cases}$$

Independent of the operational state.

Rankings of Vulnerabilities



Rankings of Vulnerabilities



- Rank the nodes: independent of the operational/synchronous state if $|\Delta\theta| < 30^\circ \rightarrow \text{LRank} \cong \text{WLRank}$.

Conclusion

Local Vulnerabilities

- Most vulnerable nodes $\rightarrow$ C_1 or C_2 (C_p) depending on τ_0 and on the dynamical variable of interest.
  $\rightarrow$ p depends on which performance measures you are interested in and on the correlation time of the perturbation.
- Resistance distance $\rightarrow$ quantify response of coupled oscillators.

Primary Control Effort for Realistic Power Networks
$\rightarrow$ MT, Coletta and Jacquod *Control Syst. Lett.* **5** (3), 929-934 (2021).
Global Robustness

$\rightarrow$ MT, Coletta and Jacquod *Phys. Rev. Lett.* **120**, 084101 (2018).
Global Robustness vs. Local Vulnerabilities

$\rightarrow$ MT and Jacquod *Phys. Rev. E* **100**, 032303 (2019).