Fault detection and probing in high-voltage power networks

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Transmission Networks
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- Faults on lines or nodes.
- Missing information about the system.
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Diffusively Coupled Systems on Complex Networks

Swing Equations in the lossless line limit (second-order Kuramoto):

\[ m_i \ddot{x}_i + d_i \dot{x}_i = \omega_i - \sum_j a_{ij} \sin(x_i - x_j) \quad i = 1, \ldots, n. \]

\[ a_{ij} = a_{ji} \geq 0. \]

**Steady-state solutions:** Synchronous state \( \{x_i^0\} \) such that:

\[ \omega_i = \sum_j a_{ij} \sin(x_i^0 - x_j^0) \quad i = 1, \ldots, n. \]

Not too heterogeneous:

\[ \omega_i = \sum_j a_{ij}(x_i^0 - x_j^0) \quad i = 1, \ldots, n. \]

\[ \rightarrow \omega = Lx^0 \quad i = 1, \ldots, n. \]
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**Perturbations**:

- at nodes \( \omega_i = \omega^0_i + \xi_n(t) \),
- on lines \( a_{ij}(t) = a^0_{ij} + \xi_l(t) \).
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Results:
If \( \max_t |\dot{\xi}_n, l(t)| \ll \min \left\{ \frac{d_i}{m_i}, \frac{\lambda_j}{\sqrt{m_i}}, \frac{\lambda_j}{d_i} \right\} \rightarrow \) Identify faulty element(s).
\( \lambda_j: \) eigenvalues of the Laplacian matrix.
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**Perturbations:**
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**Results:**
If \( \max_t |\dot{\xi}_n,1(t)| \ll \min \left\{ \frac{d_i}{m_i}, \frac{\lambda_j}{\sqrt{m_i}}, \frac{\lambda_j}{d_i} \right\} \rightarrow \) Identify faulty element(s).
\( \lambda_j: \) eigenvalues of the Laplacian matrix.

\[ \max_t |\xi_n(t)| \ll \omega_i^0, \quad \max_t |\xi_1(t)| \ll a_{ij}^0. \quad (1) \]
The Sherman-Morrison-Woodbury formula:

\[
\left( A + uv^\top \right)^{-1} = A^{-1} - \frac{A^{-1}uv^\top A^{-1}}{1 + v^\top A^{-1}u},
\]

(2)

where \( u \) and \( v \) are vectors characterizing the rank-1 perturbation.

Line perturbations: \( \tilde{L}(t) = L + \xi_l(t)e_{ij}e_{ij}^\top \)

\[ e_{ij} = (\ldots 1 \ldots -1 \ldots)^\top \]
The Kron reduction

\[ \mathbf{x} = \begin{pmatrix} x^g \\ x^c \end{pmatrix}, \quad \mathbf{\omega} = \begin{pmatrix} \omega^g \\ \omega^c \end{pmatrix}, \quad \mathbf{L} = \begin{pmatrix} L^{gg} & L^{gc} \\ L^{cg} & L^{cc} \end{pmatrix}. \] (3)

Generators:

\[ L^r = L^{gg} - L^{gc}(L^{cc})^{-1}L^{cg}. \] (4)

Natural velocities:

\[ \omega^r = \omega^g - L^{gc}(L^{cc})^{-1}\omega^c. \] (5)

Non-reduced nodes:

\[ \omega^r = L^r x^g. \] (6)

which allows to solve the equations for the subset of nodes \( I_g \).
Frequency mismatch:

\[ \psi(t) = L^r x^g(t) = L^r \left[ \tilde{L}^r(t) \right]^\dagger \tilde{\omega}^r(t), \quad (7) \]
Between non-reduced end-nodes:

\[ \tilde{\omega}^r(t) = \omega^r, \quad \tilde{L}^r(t) = L^r + \xi_1(t)e_{ij}e_{ij}^\top, \]  

(8)

Frequency mismatch:

\[
\psi(t) = L^r \left[ L^r + \xi_1(t)e_{ij}e_{ij}^\top \right]^\dagger \omega^r
\]

\[ = L^r \left[ (L^r)^\dagger - \frac{\xi_1(t)(L^r)^\dagger e_{ij}e_{ij}^\top (L^r)^\dagger}{1 + \xi_1(t)e_{ij}^\top (L^r)^\dagger e_{ij}} \right] \omega^r
\]

\[ = \omega^r - \alpha(t) \left[ e_{ij}^\top (L^r)^\dagger \omega^r \right] e_{ij}, \]

(9)
Line disturbances: Theory

**Line disturbance with at least one reduced end-node:**

\[ \psi(t) = \omega' + \tilde{\gamma}(t)\tilde{v}, \quad (10) \]

where

\[ \tilde{v} = e_i + L^{gc} (L^{cc})^{-1} e_j, \quad (11) \]
Line disturbances: Example

(b) 6; (c) 11; (d) 7-8; (e) 2-10; (f) 11-12
Line disturbances: Example

European Network:

(a) \[
\max \left| \Delta \psi(t) \right| \text{ [p.u.]} \n\]

(b) \[
\max \left| \dot{x}(t) \right| \text{ [s\^{-1}]} \n\]

(c) \[
\times 10^{-3} \max \left| \Delta z(t) \right| \text{ [p.u.]} \n\]

(d) \[
\max \left| \Delta \psi(t) \right| \text{ [p.u.]} \n\]

(e) \[
\max \left| \dot{x}(t) \right| \text{ [s\^{-1}]} \n\]

(f) \[
\max \left| \Delta z(t) \right| \text{ [p.u.]} \n\]
Line disturbances: Example

US Airports:

Multiple line disturbances [see theory in NJP 23(4), 043037 (2021)]
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Diffusively Coupled Agents

\[ \dot{x}_i(t) = \omega_i - \sum_{j} a_{ij} f_{ij}(x_i - x_j) + b_i(t), \quad i = 1, \ldots, n. \]

(12)

\[ a_{ij} > 0. \quad \text{ } \quad b_i \text{ probing/input signal.} \]

Assumption

- system stays close to a stable fixed point \( x^* \).

\[ \{x_i(t), \dot{x}_i(t)\} \rightarrow a_{ij}, n. \]
Network properties from probing

Assumption

- system stays close to a stable fixed point $x^*$.

Linearization:

$$\delta \dot{x}_i(t) = -\sum_j a_{ij} \frac{\partial f_{ij}}{\partial x}(x^0)(\delta x_i - \delta x_j) + b_i(t), \ i = 1, \ldots, n. \quad (13)$$

$$\mathbb{J}_{ij} = \begin{cases} -a_{ij} \frac{\partial f_{ij}}{\partial x}(x^0), & \text{if } i \neq j, \\ \sum_k a_{ik} \frac{\partial f_{ik}}{\partial x}(x^0), & \text{if } i = j. \end{cases} \quad (14)$$
Probing/Input signal:

\[ \delta \dot{x}_i(t) = \omega_i - \sum_j \mathbb{J}_{ij} \delta x_j + b_i(t),\ i = 1, \ldots, n. \]  

\[ b_i(t) = b_0 \sin(\omega_0 t) \]

\[ \{x_i(t), \dot{x}_i(t), \ldots\} \]
Number of Nodes

**Time-scales:**

Eigenvalues of $\mathbb{J}$: $\{\lambda_1, \ldots, \lambda_n\}$.

$\lambda_1 = 0 \rightarrow u_{1,i} = (1, \ldots, 1)/\sqrt{n}$.

- fast-varying probing $\rightarrow$ remains local.
- slow-varying probing $\rightarrow$ spreads in the whole system.
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$$\delta x_i(t) = \frac{b_0}{n\omega_0} [1 - \cos (\omega_0 t)] + \mathbb{J}_a^\dagger b_0 \sin (\omega_0 t), \quad (16)$$

$$\hat{n} = \frac{b_0}{\omega_0 \delta}. \quad (17)$$
Number of Nodes

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Missing information about the system.


CCS Satellite

CCS2021 Satellite Symposium

Data-based Diagnosis of Networked Dynamical Systems
Wednesday October 27th, 2021

Complex networks of dynamical agents are widely used to model the behavior of large physical or virtual systems. Unfortunately, due to the often abstract nature of such networks or the size thereof, it is sometimes difficult to assess correctly their structure and parameters. With the ever increasing amount of data accessible nowadays, it is natural to attempt to recover structural information of the system from measurements.

Altogether, there are two overlapping questions that we would like to treat in this symposium:

- What networks characteristics can be recovered from time-series measurements of its agents?
- How to identify and locate disturbances from time-series measurements?

Confirmed speakers

- Misha Chertkov, University of Arizona, USA.
- Pietro De Lellis, University of Naples Federico II, Italy.
- Philippe Jacquod, University of Geneva, Switzerland.
- Nathan Kutz, University of Washington, USA.
- Andrey Lokhov, Los Alamos National Laboratory, USA.
- Enrique Mallada, Johns Hopkins University, USA.
- Edward Ott, University of Maryland, USA.
- Tiago de Paula Peixoto, Central European University, Austria-Hungary.
- Leonardo Rydin Gorjão, Oslo Metropolitan University, Norway.
- Anna Scaglione, Arizona State University, USA.
- Marc Timme, Technische Universität Dresden, Germany.
- Melvyn Tyloo, University of Geneva, Switzerland.
- Marc Vuffray, Los Alamos National Laboratory, USA.
- Gil Zussman, Columbia University, USA.