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IEEE Access 10, 65118 - 65125 (2022).

Toward Model Reduction for Power System Transients With Physics-Informed PDE

LAURENT PAGNIER[1], (Member, IEEE),
JULIAN FRITZSCH[2,3], (Graduate Student Member, IEEE),
PHILIPPE JACQUOD[2,3], (Member, IEEE), AND
MICHAEL CHERTKOV[1], (Senior Member, IEEE)

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Construction of a reduced, continuous model

- Reproduces steady-state

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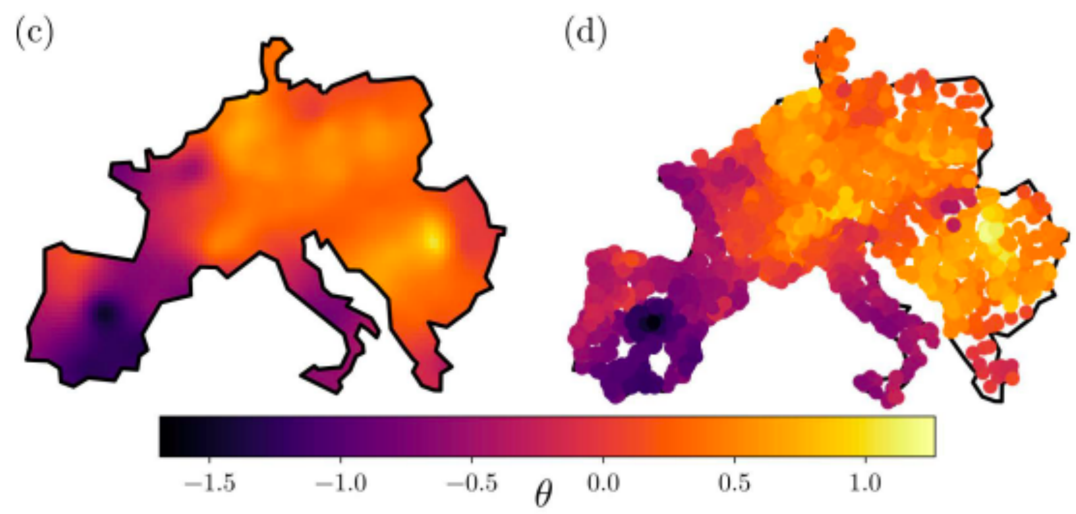

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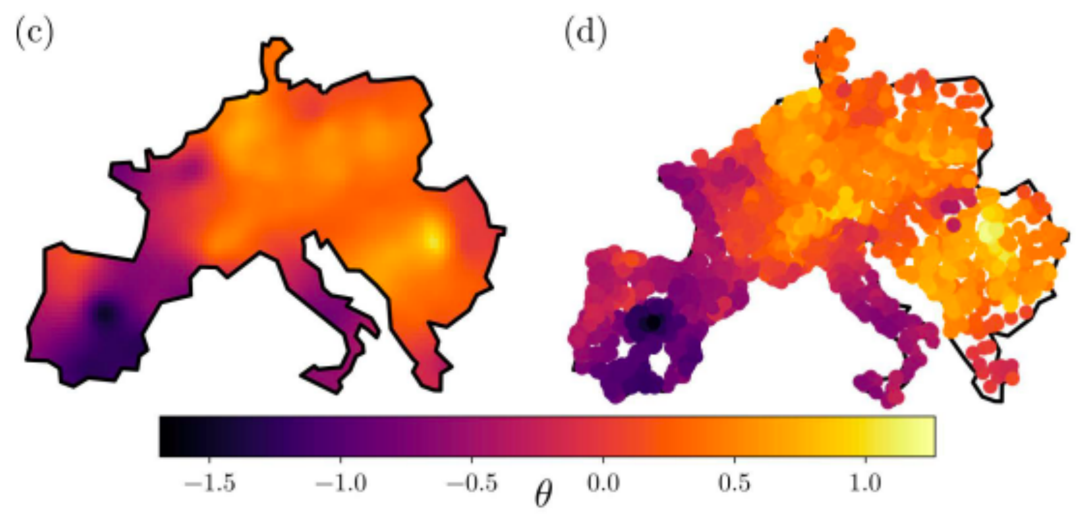

- Captures dynamics

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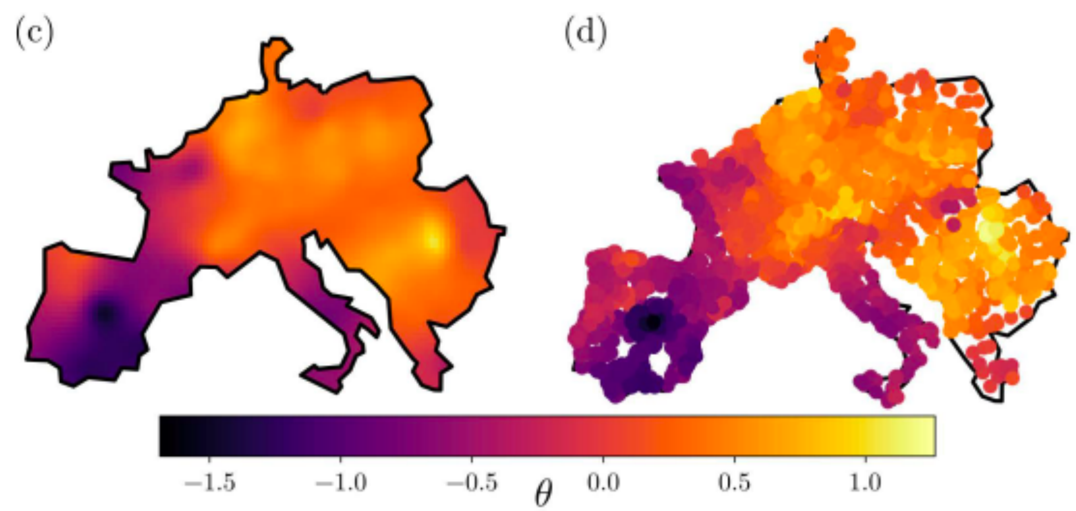

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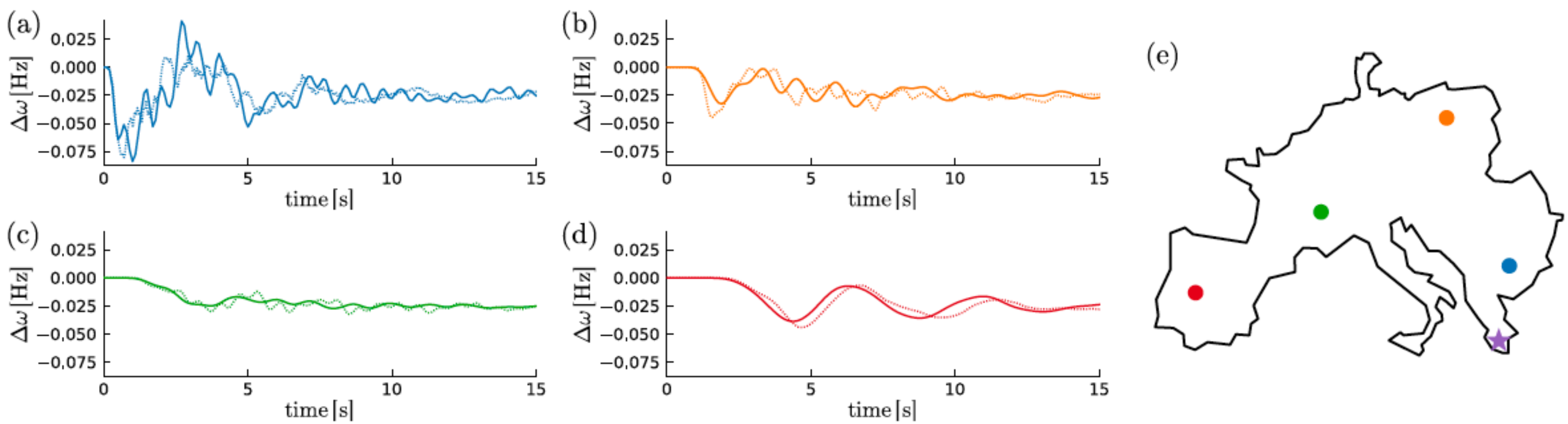

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Construction of a reduced, continuous model

- Reproduces steady-state
- Captures dynamics

In progress

Extraction of real-time dynamical parameters

- Estimate of available ancillary resources

Reconstructing Networks from Partial Measurements

Philippe Jacquod
CCS2023 Satellite Symposium

Colls.: R. Delabays (HES-SO)
M. Tyloo (LANL)

M Tyloo, R Delabays, and PJ, Chaos 31, 103117 (2021)

UNIVERSITÉ DE GENÈVE
FACULTÉ DES SCIENCES

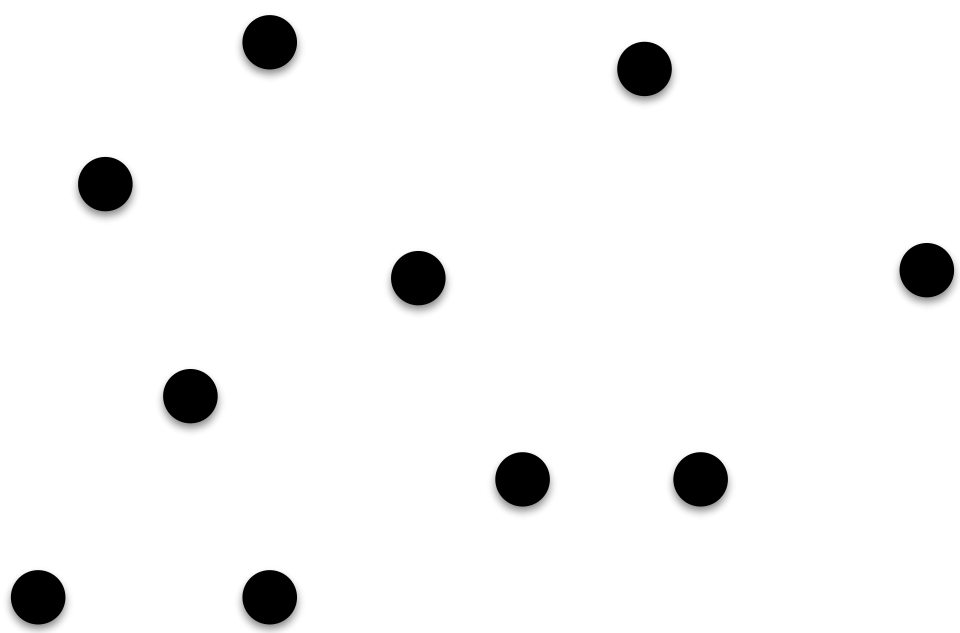
Swiss National Science Foundation

Hes·so // VALAIS WALLIS
School of Engineering

The problem

* agents
* their degrees of freedom

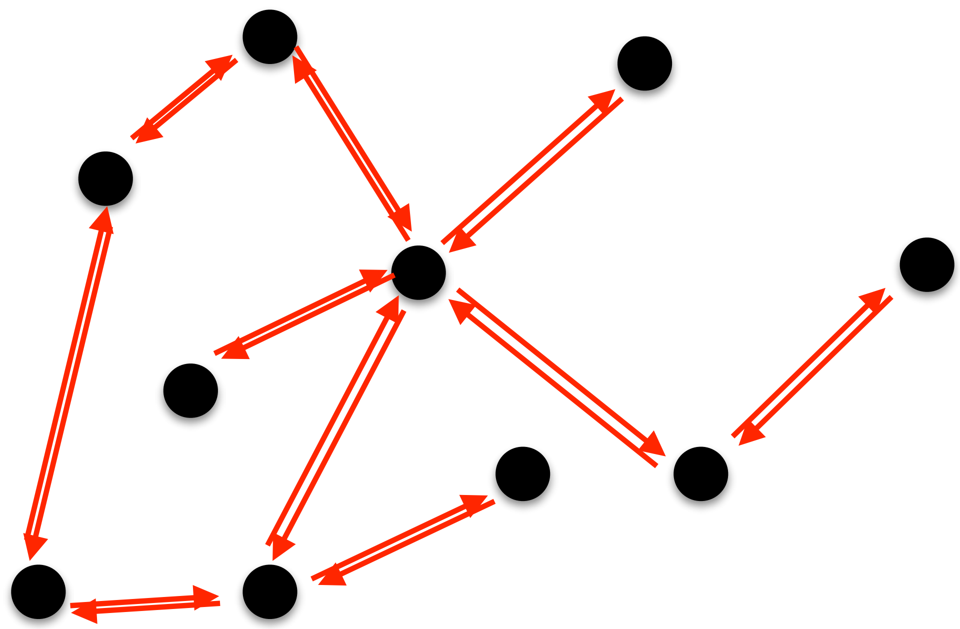
$$\{x_i(t), \dot{x}_i(t)\}$$



The problem

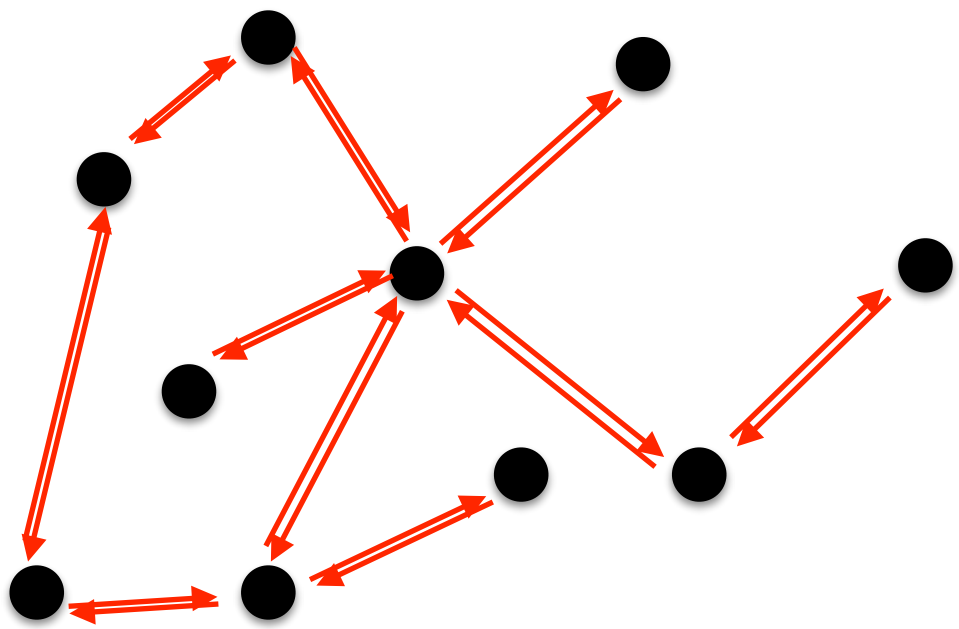
* n agents
* their degrees of freedom
* what can we know of the way they interact ?

$$\{x_i(t), \dot{x}_i(t)\}$$



The problem

$\{x_i(t), \dot{x}_i(t)\}$



What we want to extract :

* Number n of agents ?
* Connectivity ? Graph topology ?

From
-complete / partial
-active / passive
measurements.

Previous works

* Probing, i.e. injecting controlled signal and measuring the response

D. Yu, M. Righero, and L. Kocarev, PRL 2006
D. Yu and U. Parlitz, EPL 2008
M. Tyloo and R. Delabays, J Phys Complex 2021

* Optimization of likelihood cost function

D.-T. Hoang, J. Jo, and V. Periwal, PRE 2019
V.A.Makarov, F. Panetsos, and O. de Febo, J. Neurosci. Methods 2005
M. J. Panaggio, M.-V. Ciocanel, L. Lazarus, C. M. Topaz, and B. Xu, Chaos 2019

* Short-time dynamics / trajectory correlations

R. Dahlhaus, M. Eichler, and J. Sandkühler, J. Neurosci. Methods 1997
K. Sameshima and L. A. Baccalá, J. Neurosci. Methods 1999
M.E.J. Newman, Nat. Phys. 2018
T. P. Peixoto, PRL 2019
M. G. Leguia, C. G. B. Martínez, I. Malvestio, A. T. Campo, R. Rocamora,
Z. Levnaji´c, and R. G. Andrzejak, PRE 2019
A. Banerjee, J. Pathak, R. Roy, J. G. Restrepo, and E. Ott, Chaos 2019

Noise vs. frequency correlators vs. connectivity

* Two-point correlators

$$C_{ij} = \langle x_i(t) x_j(t) \rangle = \lim_{T\to\infty} \frac{1}{T} \int_0^T x_i(t) x_j(t) \, \mathrm{d}t$$

J. Ren, W.-X. Wang, B. Li, and Y.-C. Lai, PRL 2010
W.-X. Wang, J. Ren, Y.-C. Lai, and B. Li, Chaos 2012
E. S. C. Ching and H. C. Tam, PRE 2017
Y. Chen, S. Wang, Z. Zheng, Z. Zhang, and G. Hu, EPL 2016
H. C. Tam, E. S. C. Ching, and P.-Y. Lai, Physica A 2018

$$\boldsymbol{C} \propto \boldsymbol{L}^{\dagger}$$

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Reconstruction of the network Laplacian matrix via inversion of the equal time, 2-point correlation matrix

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- Either you have the full matrix, i.e. from a complete measurement, or you have nothing.

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Reconstruction of the network Laplacian matrix via inversion of the equal time, 2-point correlation matrix

- Either you have the full matrix, i.e. from a complete measurement, or you have nothing.

- What can we do if we access only to a subset of all agents ?

Earlier works

PHYSICAL REVIEW LETTERS **120**, 084101 (2018)

Robustness of Synchrony in Complex Networks and Generalized Kirchhoff Indices

M. Tyloo,[1,2] T. Coletta,[1] and Ph. Jacquod[1]
[1]*School of Engineering, University of Applied Sciences of Western Switzerland HES-SO, CH-1951 Sion, Switzerland*
[2]*Institute of Physics, EPF Lausanne, CH-1015 Lausanne, Switzerland*

Trace of frequency correlation matrix = trace of graph Laplacian
Trace of position correlation matrix = trace of inverse Laplacian

Earlier works

PHYSICAL REVIEW LETTERS 120, 084101 (2018)

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Trace of position correlation matrix = trace of inverse Laplacian

SCIENCE ADVANCES | RESEARCH ARTICLE

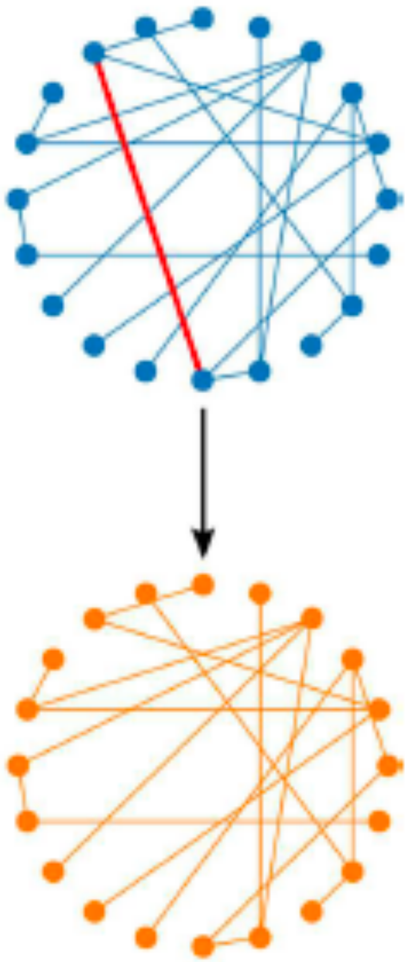
APPLIED SCIENCES AND ENGINEERING

The key player problem in complex oscillator networks and electric power grids: Resistance centralities identify local vulnerabilities

M. Tyloo[1,2], L. Pagnier[1,2], P. Jacquod[2,3*]

Diagonal frequency correlators = diagonal elements of graph Laplacian
Diagonal position correlators = diagonal elements of inverse Laplacian

Noise vs. frequency correlators vs. connectivity



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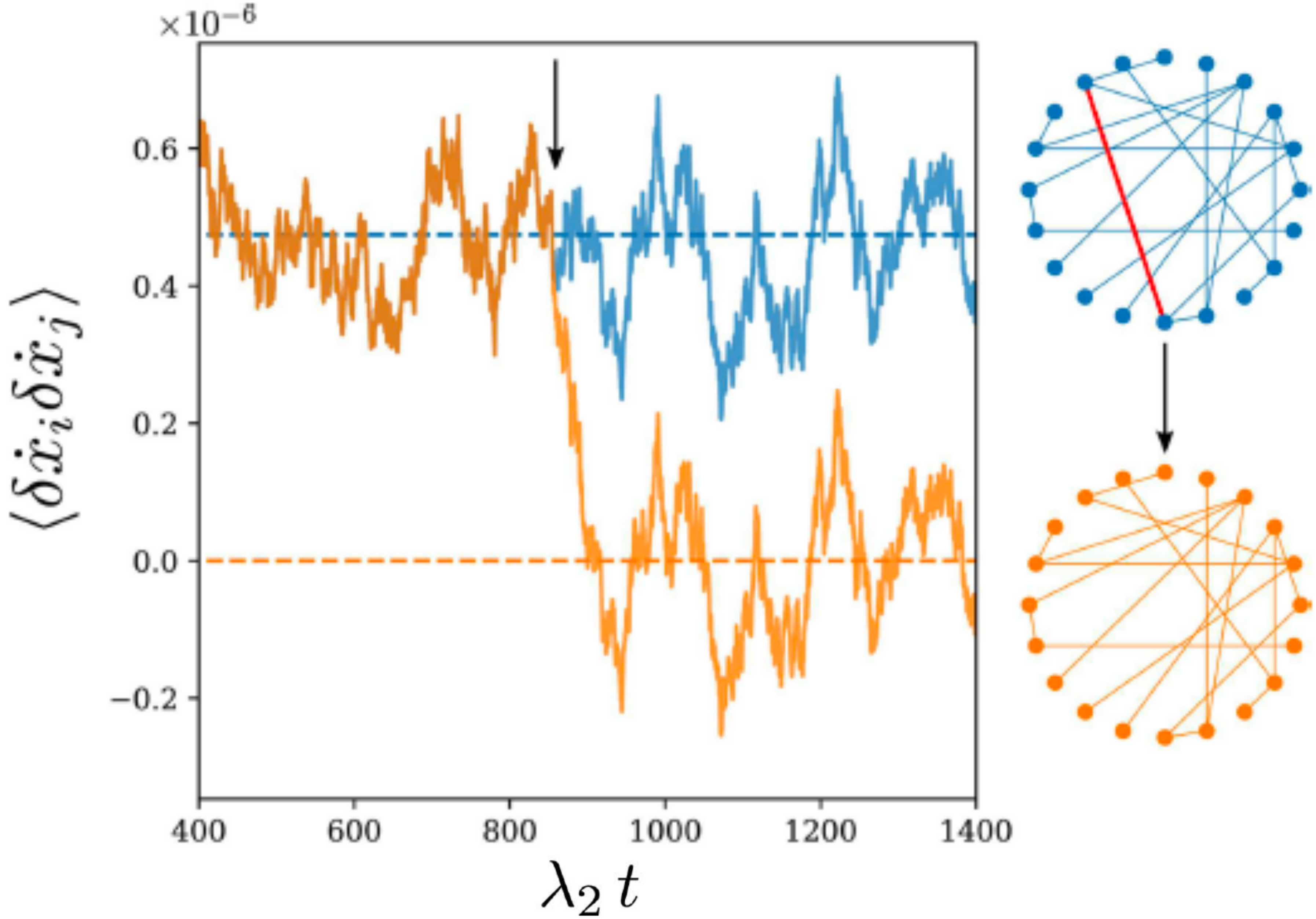

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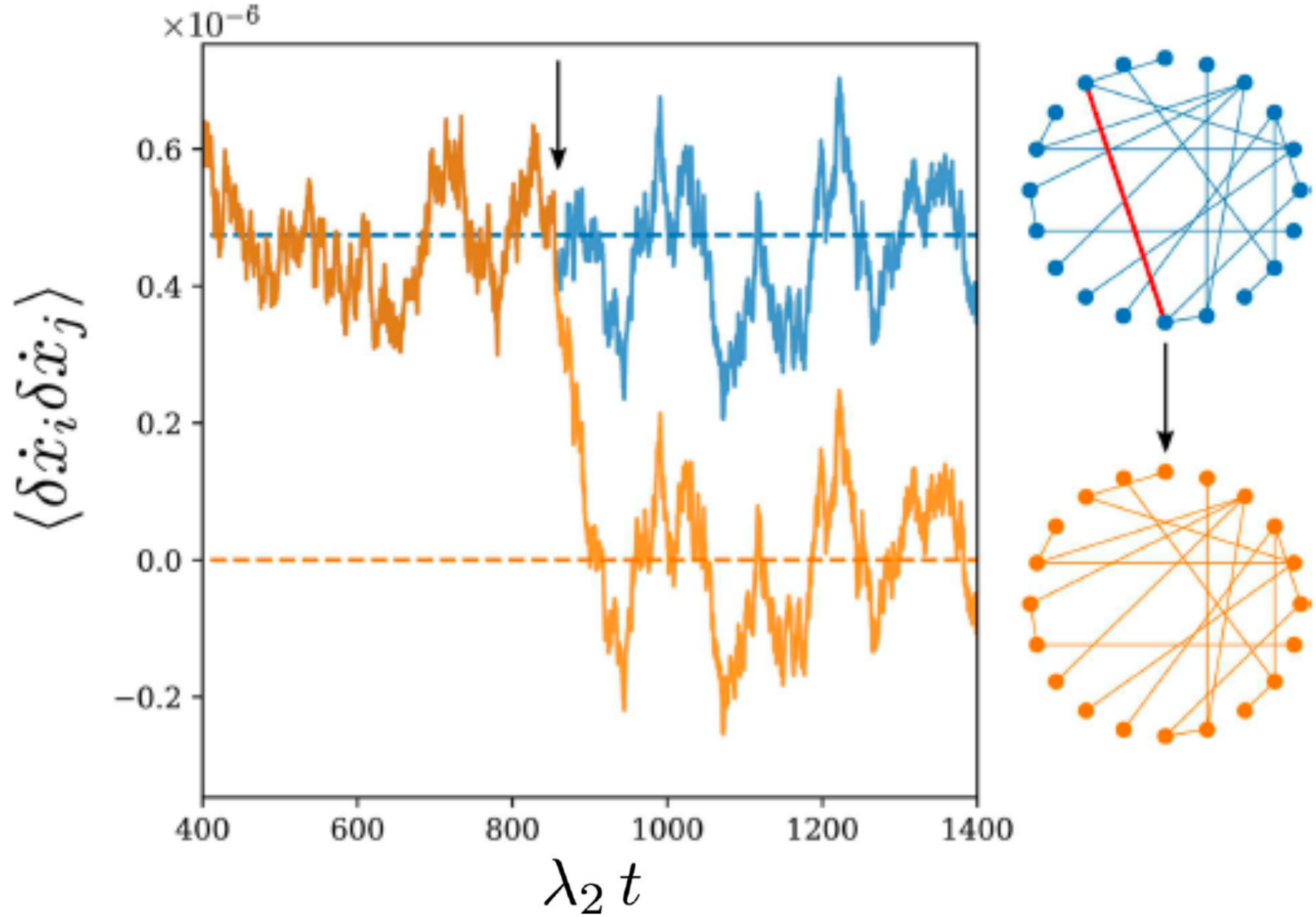
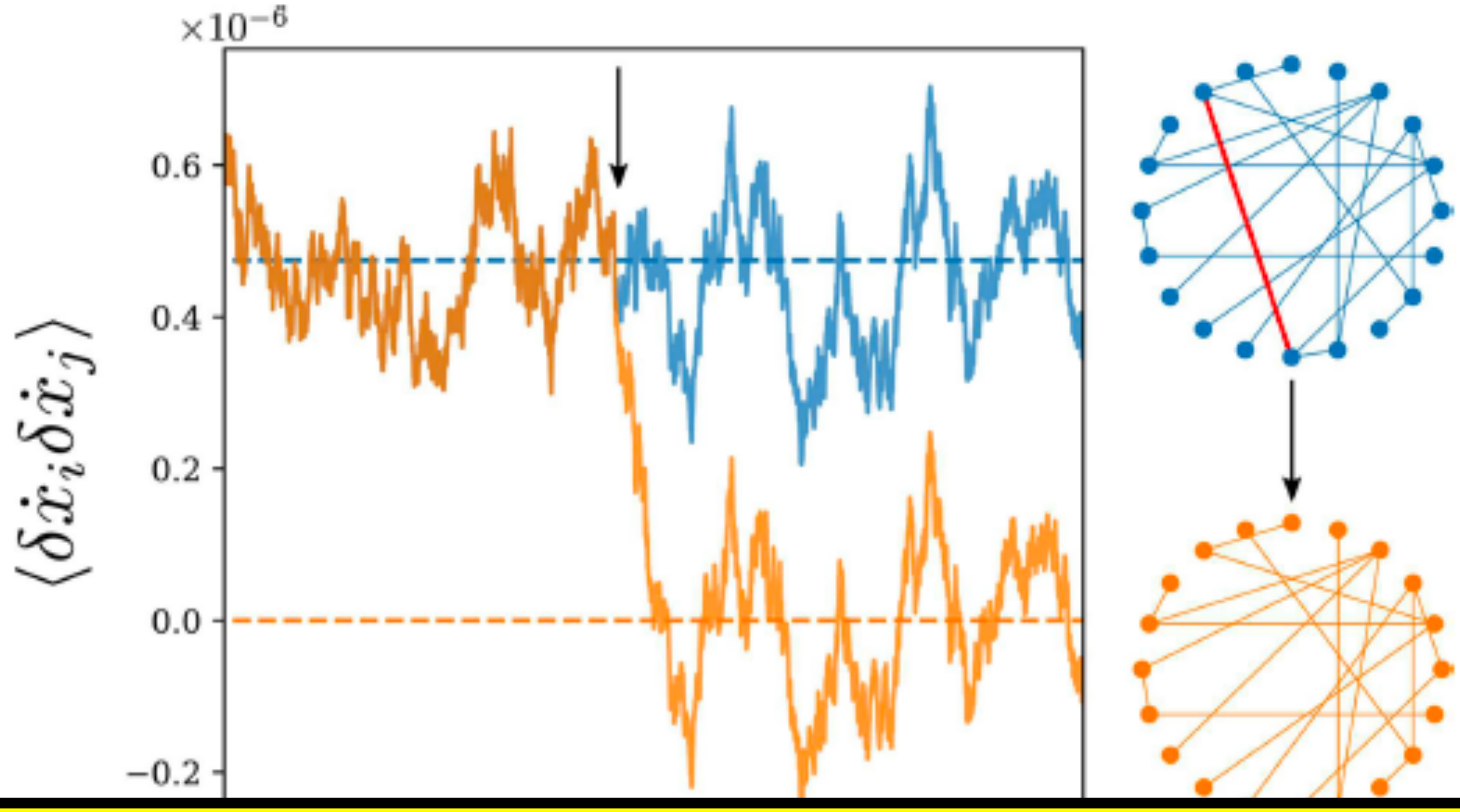


$$\langle \delta\dot{x}_i \delta\dot{x}_j \rangle = \xi_0^2 \sum_{k=q}^{\infty} (-\tau_0)^k \left(\mathbb{J}^k\right)_{ij}$$

Noise vs. frequency correlators vs. connectivity



-direct extraction of Laplacian
-partial inference from partial measurements

$$\langle \delta\dot{x}_i \delta\dot{x}_j \rangle = \xi_0^2 \sum_{k=q}^{\infty} (-\tau_0)^k \left(\mathbb{J}^k\right)_{ij}$$

M Tyloo, R Delabays, and PJ, Chaos 31, 103117 (2021)

Sketch of the analytics (i)

The model

Unperturbed dynamics $\dot{\mathbf{x}}(t) = \mathbf{F}[\mathbf{x}(t)]$ $\mathbf{F}[\mathbf{x}^*] = 0$

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Linearization about steady-state + perturbation

$$\boldsymbol{\delta\dot{x}} = -\mathbb{J}(\boldsymbol{x}^*)\,\boldsymbol{\delta x} + \boldsymbol{\xi}$$

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Network/coupling structure

$$\mathbb{J}_{ij}(\boldsymbol{x}^*) = -\partial F_i(\boldsymbol{x}^*)/\partial x_j$$

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Unperturbed dynamics $\dot{\mathbf{x}}(t) = \mathbf{F}[\mathbf{x}(t)]$ $\mathbf{F}[\mathbf{x}^*] = 0$

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Network/coupling structure

$$\mathbb{J}_{ij}(\boldsymbol{x}^*) = -\partial F_i(\boldsymbol{x}^*)/\partial x_j$$

- $\mathbb{J}(\boldsymbol{x}^*)$ is symmetric and positive semidefinite (undirected coupling; stable fixed point)

Sketch of the analytics (ii)

Modal decomposition of J

$$\boldsymbol{\delta}\dot{\boldsymbol{x}} = -\mathbb{J}(\boldsymbol{x}^*)\,\boldsymbol{\delta x} + \boldsymbol{\xi}$$

Real eigenvalues $0 \leq \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$

Orthogonal eigenbasis $\{\boldsymbol{u}_\alpha\}_{\alpha=1}^{n}$

$$\boldsymbol{\delta x}(t) = \sum_\alpha c_\alpha(t)\boldsymbol{u}_\alpha$$

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- Langevin equation for expansion coefficients

- Solutions

$$\dot{c}_\alpha(t) = -\lambda_\alpha c_\alpha(t) + \mathbf{u}_\alpha \cdot \boldsymbol{\xi}(t)$$

$$c_\alpha(t) = e^{-\lambda_\alpha t}\int_0^t e^{\lambda_\alpha t}\mathbf{u}_\alpha \cdot \boldsymbol{\xi}(t')\mathrm{d}t'$$

- Velocity correlator

$$\langle \delta\dot{x}_i(t)\delta\dot{x}_j(t)\rangle = \sum_{\alpha,\beta}\langle \dot{c}_\alpha(t)\dot{c}_\beta(t)\rangle u_{\alpha,i}u_{\beta,j}$$

Sketch of the analytics (iii)

$$\boldsymbol{\delta \dot{x}} = -\mathbb{J}(\boldsymbol{x}^*)\,\boldsymbol{\delta x} + \boldsymbol{\xi}$$

Two-point velocity correlators

Need to define first and second moment of noise
-> Orstein-Uhlenbeck

$$\langle \xi_i(t) \rangle = 0 \qquad \langle \xi_i(t+\Delta t/2)\xi_j(t-\Delta t/2)\rangle = \xi_0^2\, \delta_{ij} \exp\left(-|\Delta t|/\tau_0\right)$$

$$\lim_{t\to\infty} \langle \delta\dot{x}_i(t)\delta\dot{x}_j(t)\rangle = \xi_0^2 \left(\delta_{ij} - \sum_\alpha u_{\alpha,i} u_{\alpha,j} \frac{\lambda_\alpha \tau_0}{1+\lambda_\alpha \tau_0} \right)$$

Note : $\langle \cdots \rangle = \lim_{T\to\infty} T^{-1} \int_0^T \cdots \, \mathrm{d}t$

Sketch of the analytics (iv) - noise with short correlation time

$$\delta \dot{x} = -\mathbb{J}(x^*)\,\delta x + \xi$$

Two-point velocity correlators

Need to define first and second moment of noise
 -> Orstein-Uhlenbeck

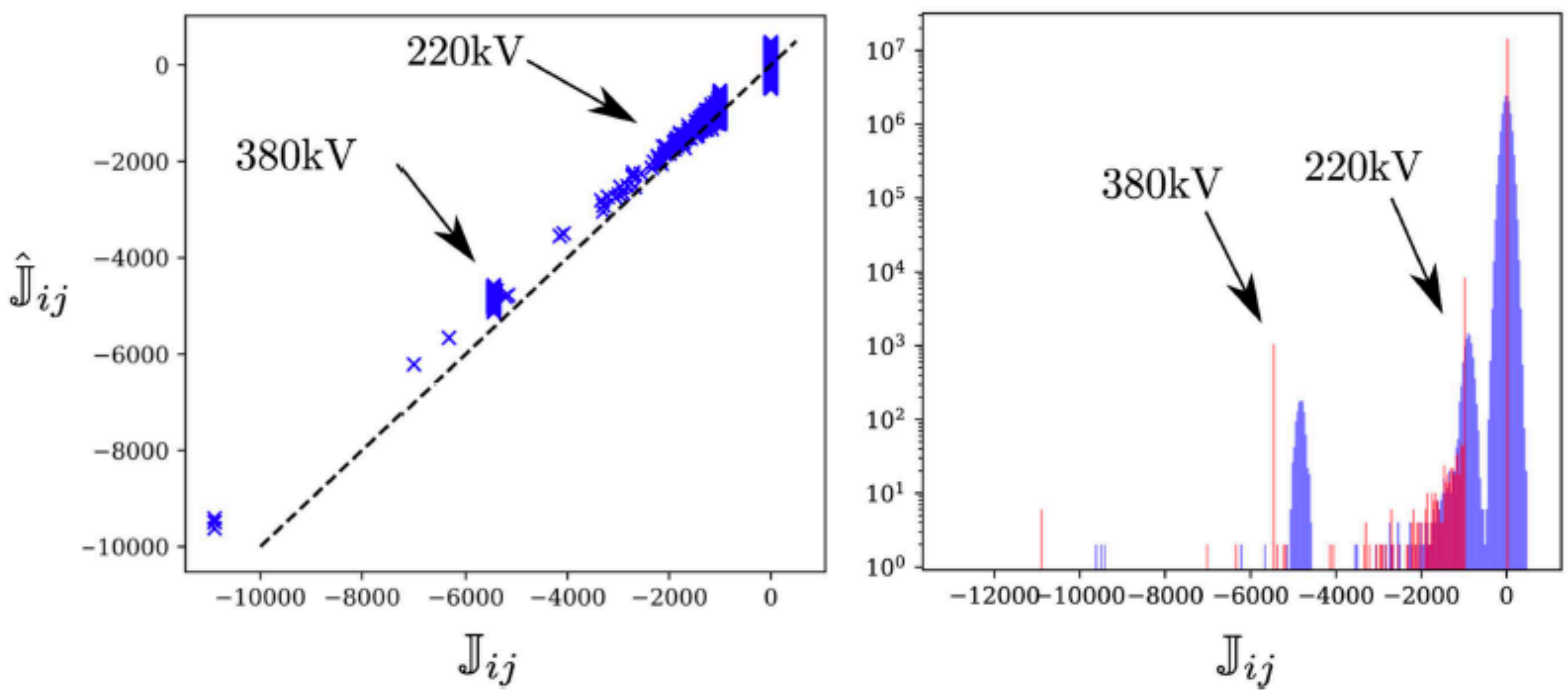
$$\langle \xi_i(t) \rangle = 0 \qquad \langle \xi_i(t+\Delta t/2)\xi_j(t-\Delta t/2) \rangle = \xi_0^2\,\delta_{ij}\exp\left(-|\Delta t|/\tau_0\right)$$

With short correlation time :

$$\lim_{t\to\infty} \langle \delta\dot{x}_i(t)\delta\dot{x}_j(t) \rangle = \xi_0^2 \left[\delta_{ij} + \sum_{k=1}^{\infty} (-\tau_0)^k (\mathbb{J}^k)_{ij} \right]$$

Note : $\langle \cdots \rangle = \lim_{T\to\infty} T^{-1} \int_0^T \cdots \,\mathrm{d}t$

Direct reconstruction

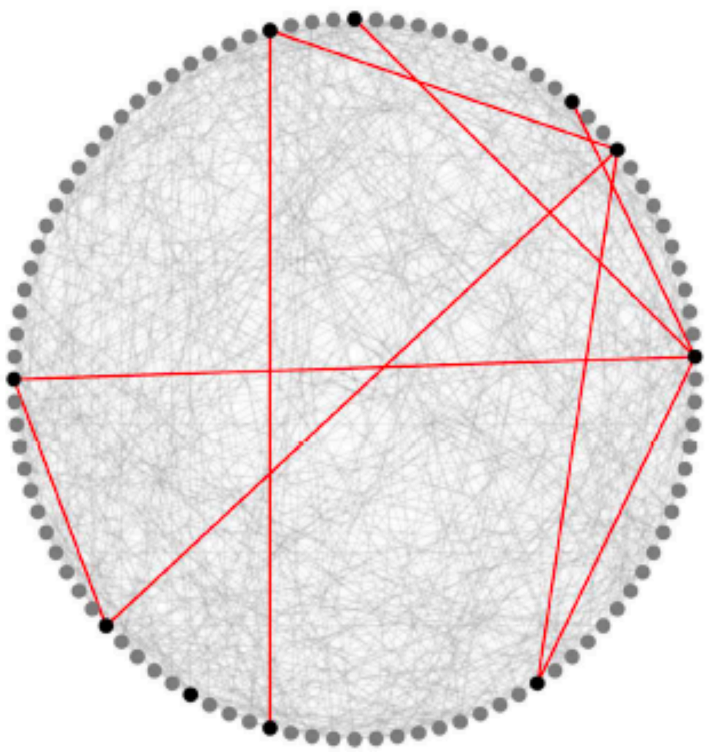


Relatively short correlation time

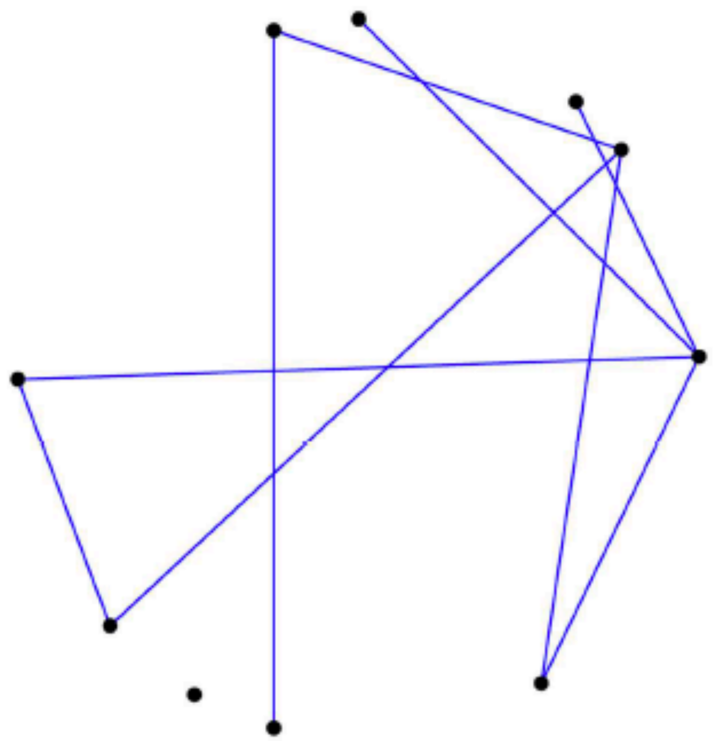
$$\hat{\mathbb{J}}_{ij} = \left(\delta_{ij} - \langle \delta \dot{x}_i \delta \dot{x}_j \rangle / \xi_0^2\right) \tau_0^{-1}$$

L. Pagnier and P. Jacquod, PanTaGruEl, Zenodo Repository (2019). doi.org/10.5281/zenodo.2642175

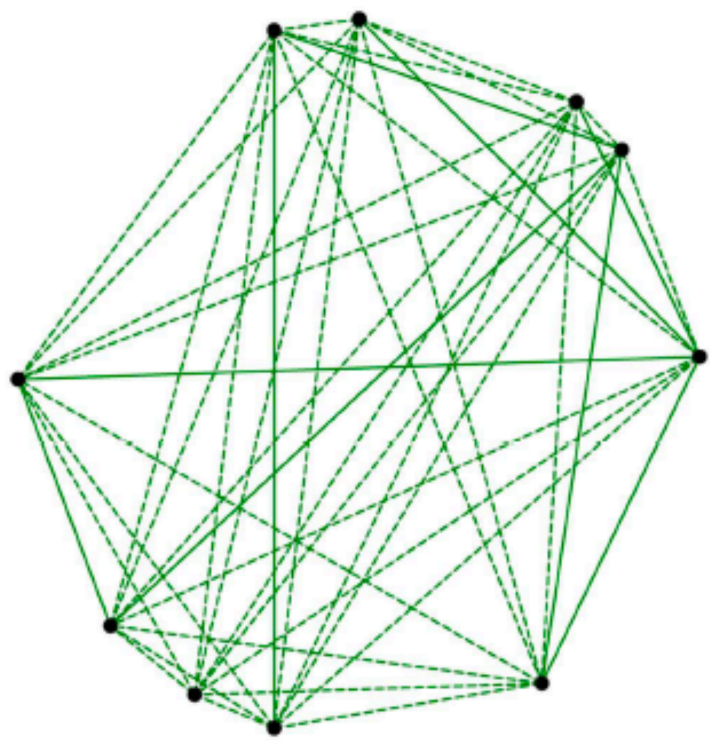
Partial reconstruction (i) : n=100 Erdös-Rényi



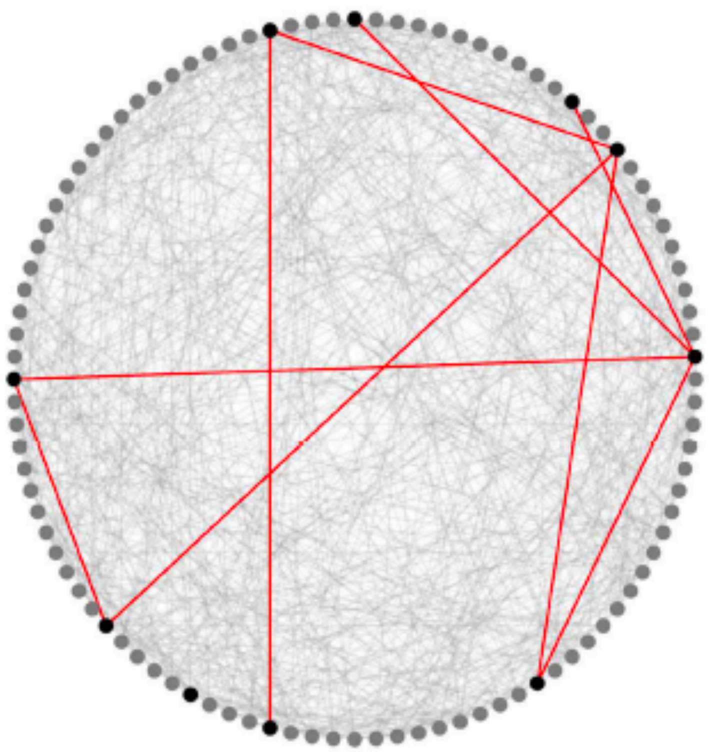
Velocity correlators



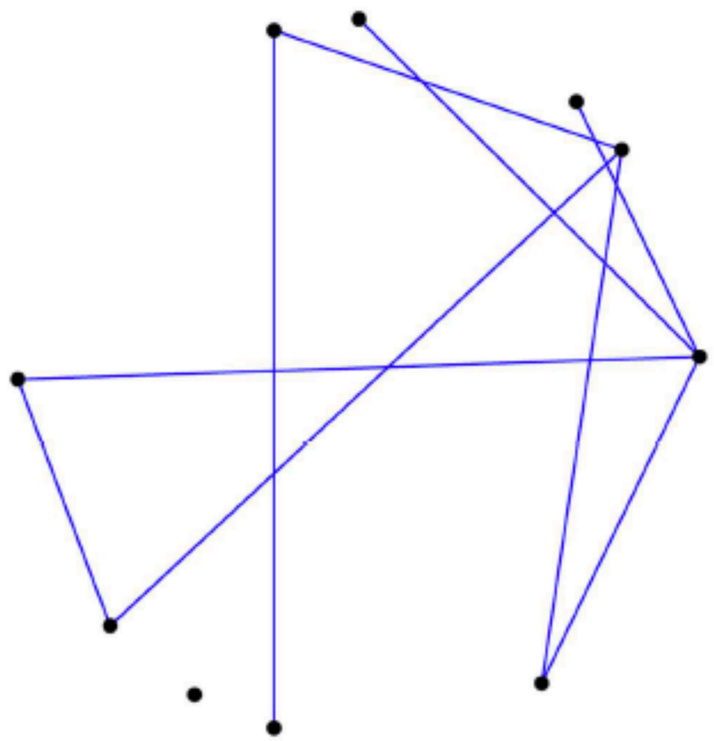
Position correlators



Partial reconstruction (i) : n=100 Erdös-Rényi



Velocity correlators



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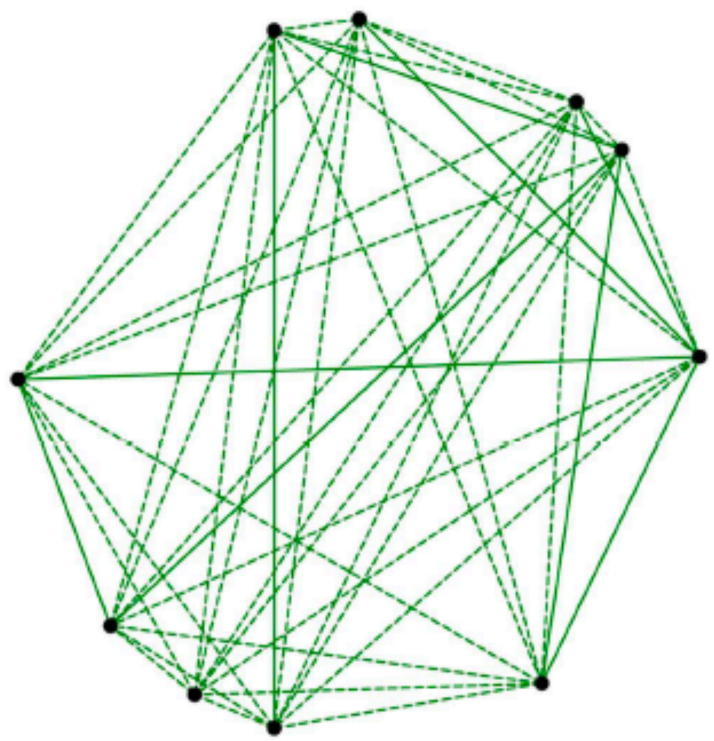
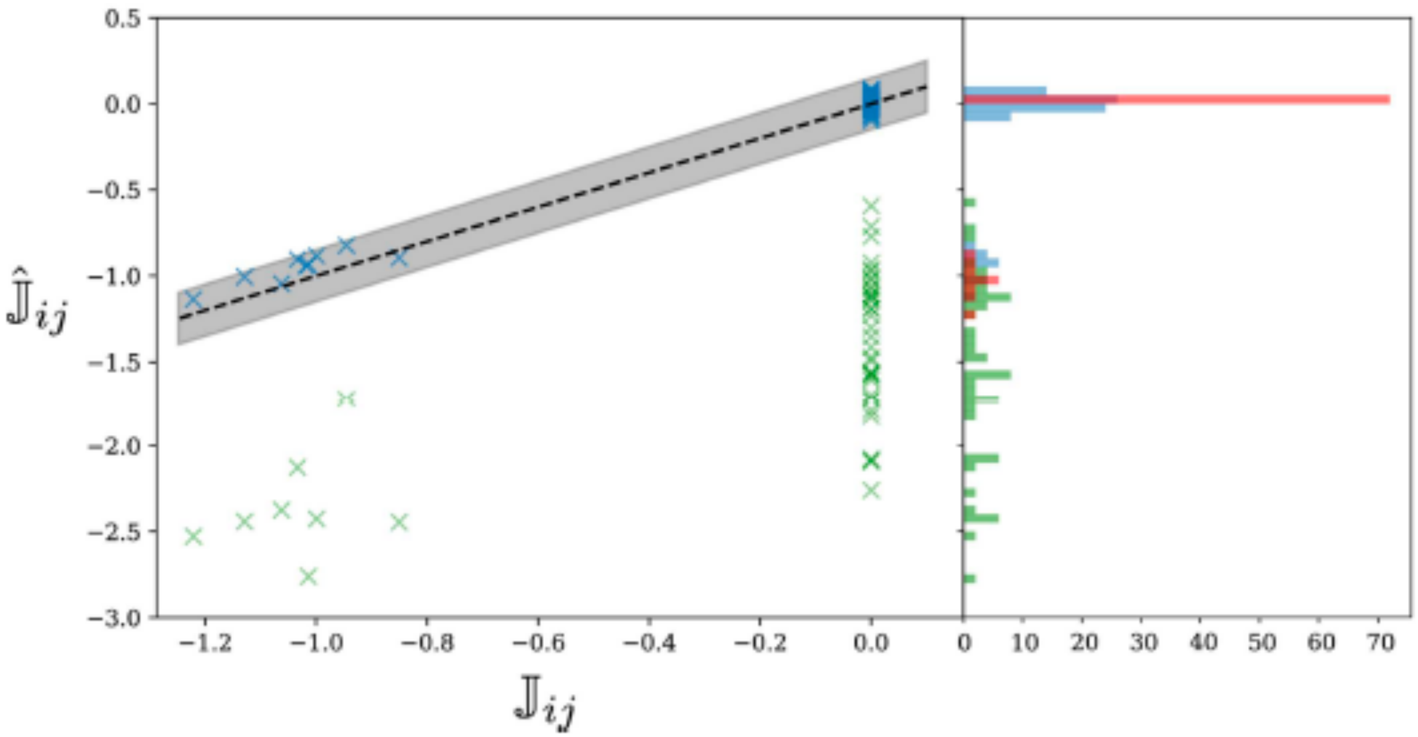
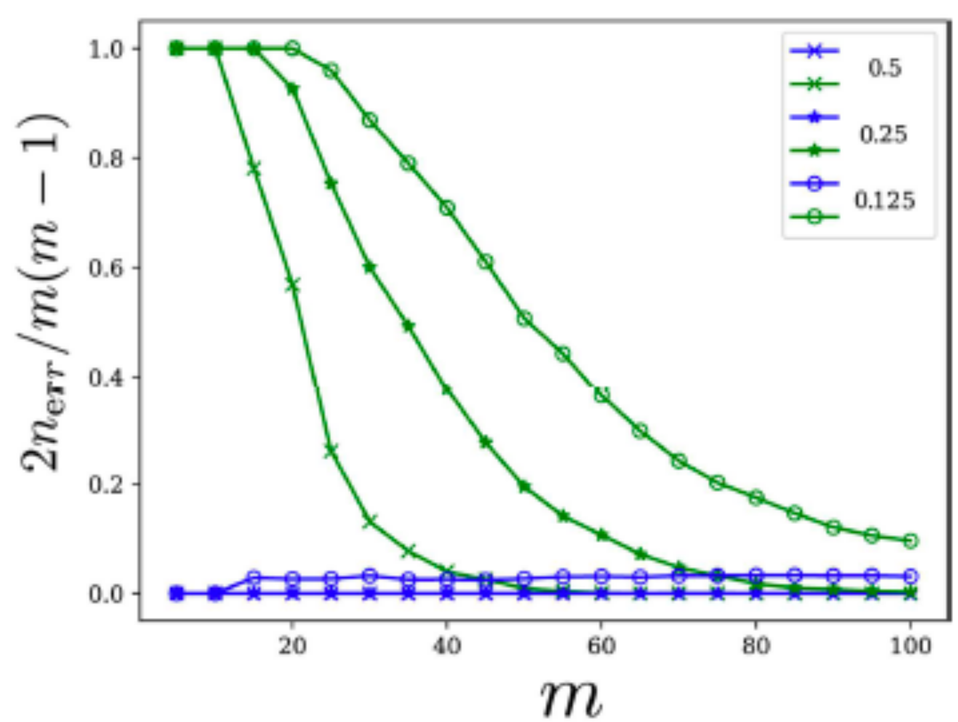
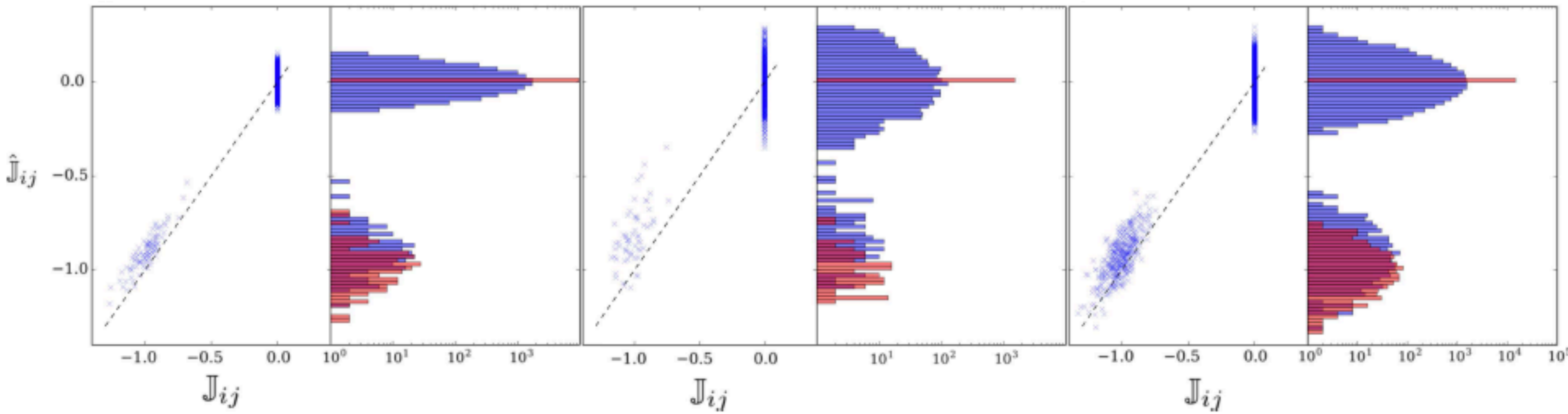




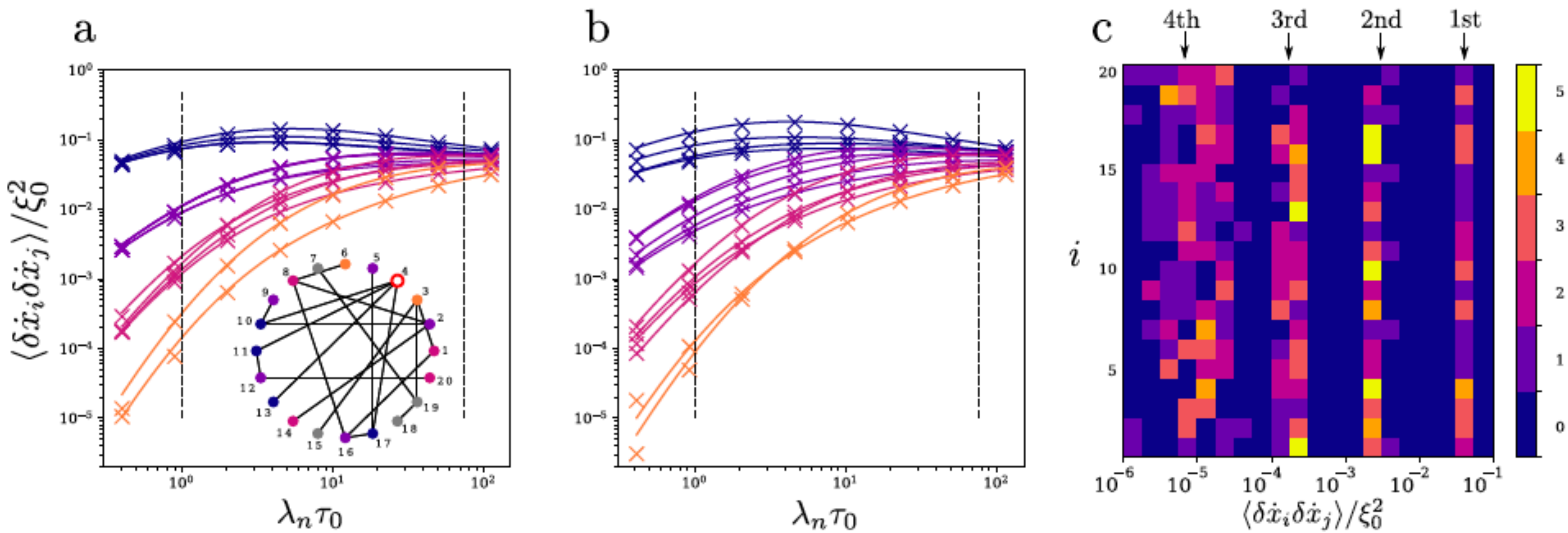


Partial reconstruction (ii) : n=1000 with m=100 observable



Erdös-Rényi Barabasi-Albert Watts-Strogatz

Geodesic distance



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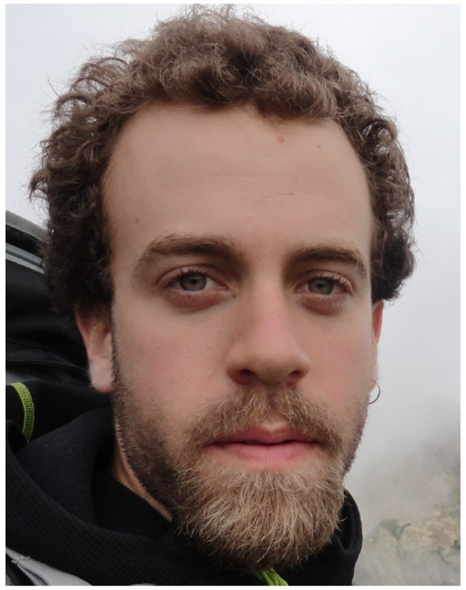
Thank U's

Laurent Pagnier
U of Arizona

Melvyn Tyloo
LANL

Tommaso Coletta
Sophia Genetics



Robin Delabays
HES-SO

Swiss National
Science Foundation