Disruption of Kuramoto Networks

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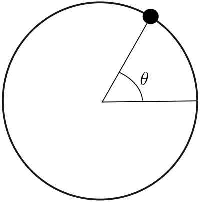
Phase oscillators

Single phase oscillator: $\dot{\theta} = \omega$



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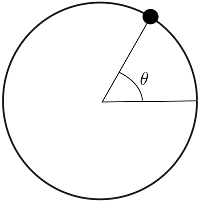
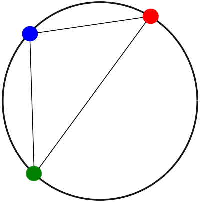


Coupled phase oscillators: $\dot{\theta}_i = \omega_i - \sum_j a_{ij} f(\theta_i - \theta_j)$



Phase oscillators

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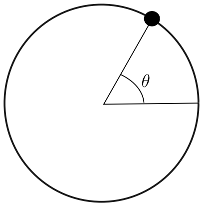

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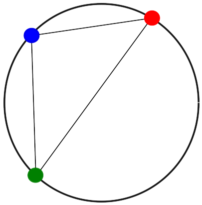

Synchronization: phase-locked $\dot{\theta}_i(t) = \dot{\theta}_j(t)\,,\ \forall i, j\,.$

Synchronization

- Simple experiment: coupled metronomes
- Other examples:
  - Fireflies flashing in unison
  - People clapping their hands or walking on a bridge
  - Synchronization of the quantum phase of Josephson junctions arrays
  - Synchronization of the phase of the voltages in electric power grids
  - Consensus formation on influence networks
  - Vehicular platoon formation
  - Orchestra

Modelling of Synchronization

Kuramoto model

$$\dot{\theta}_i = \omega_i - \frac{K}{N}\sum_{j=1}^{N} \sin(\theta_i - \theta_j)\,, \text{for } i = 1, ..., N\,. \tag{1}$$

ω_i: natural frequencies.

Y. Kuramoto, Lecture Notes in Physics 39, International Symposium on Mathematical Problems in Theoretical Physics (1975).

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Order parameter

$$re^{i\,\psi} = N^{-1}\sum_{j=1}^{N} e^{i\,\theta_j} \tag{2}$$

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$$re^{i\,\psi} = N^{-1}\sum_{j=1}^{N} e^{i\,\theta_j} \tag{2}$$

ψ: average phase.

Illustration

$$\dot{\theta}_i = \omega_i - Kr\sin(\psi - \theta_i)\,, \text{for } i = 1, ..., N\,. \tag{3}$$

Y. Kuramoto, Lecture Notes in Physics 39, International Symposium on Mathematical Problems in Theoretical Physics (1975).

Dante R. Chialvo

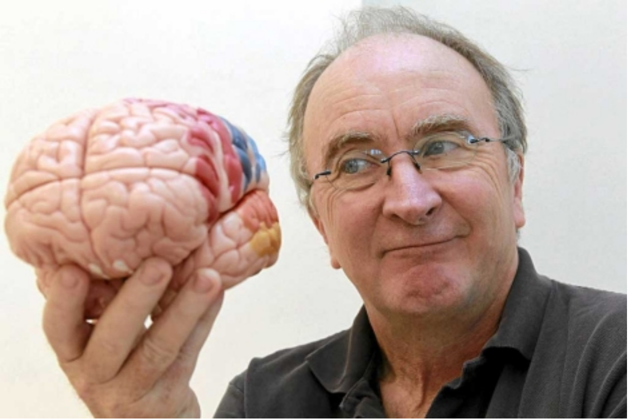

Figure: https://www.quantamagazine.org/toward-a-theory-of-self-organized-criticality-in-the-brain-20140403/

During a discussion between ABQ and SF: "...in the brain, nothing is in a synchronized state!"

Coupled Oscillators: Power grids

Second order Kuramoto model

$$m_i\ddot{\theta}_i + d_i\dot{\theta}_i = P_i - \sum_j a_{ij}\sin(\theta_i - \theta_j) \ \ , \ i = 1, ..., n.$$

$a_{ij} = a_{ji} \geq 0$.
P_i : natural frequencies.
m_i : inertia.
d_i : damping.

Electric Power Network (in the lossless line approximation)
P_i : injected/consumed power.
$m_i = 0$: loads.
$m_i \neq 0$: generators.
$a_{ij}\sin(\theta_i - \theta_j)$: power flow from i to j .

J. A. Acebrón, L. L. Bonilla, Conrad J. Pérez Vicente, F. Ritort, and R. Spigler, Rev. Mod. Phys. **77**, 137 (2005)

Power system control and stability PM Anderson, AA Fouad - 1977

Synchronization on networks

Kuramoto model

$$\dot{\theta}_i = \omega_i - \sum_{j=1}^{N} a_{ij} \sin(\theta_i - \theta_j) \,, \text{for } i = 1, ..., N \,. \tag{4}$$

ω_i: natural frequencies.
a_{ij}: adjacency matrix.

Good survey Dörfler and Bullo, Automatica **50** (6), 1539-1564, (2014).

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Multistability

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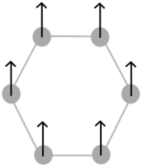

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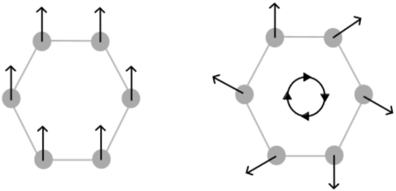

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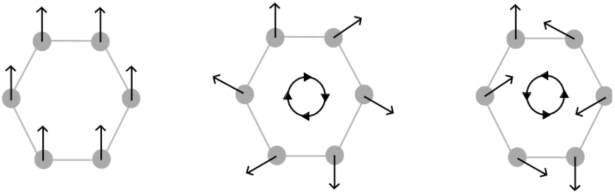

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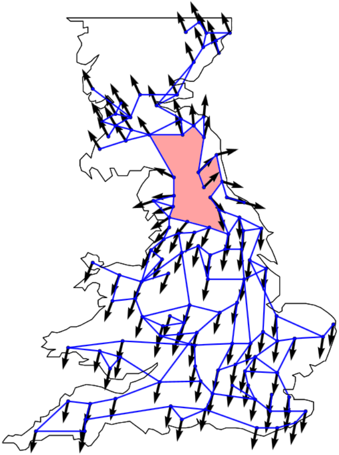
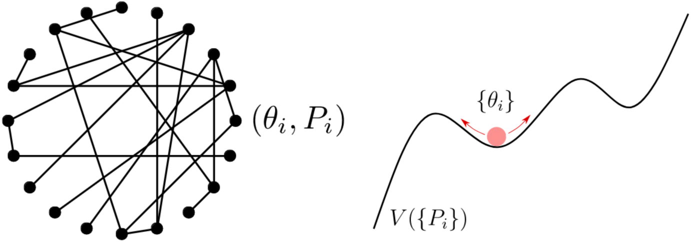


Delabays, MT, Jacquod, Chaos **27**(10), 103109 (2017)

Robustness of synchronous networks



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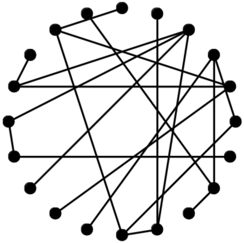
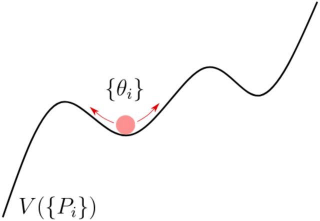
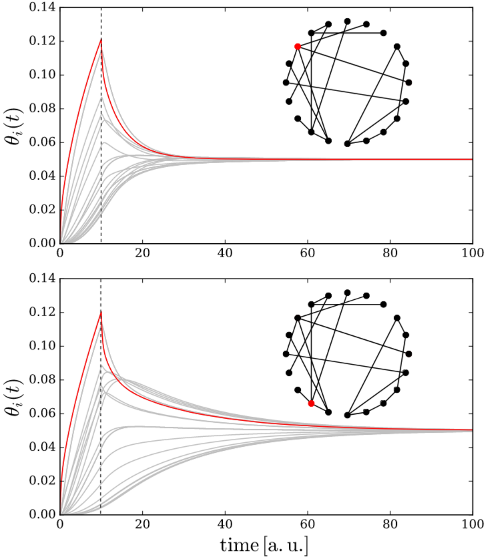




- Size of the basin of attraction
- Near equilibrium dynamics
- Transitions between fixed points

Robustness of synchronous networks



Robustness of synchronous networks to noise inputs

Near equilibrium dynamics

$$0 = \omega_i - \sum_{j=1}^{N} b_{ij} \sin(\theta_i^{(0)} - \theta_j^{(0)})\,, \text{for } i = 1, ..., N\,. \tag{5}$$

Robustness of synchronous networks to noise inputs

Near equilibrium dynamics

$$0 = \omega_i - \sum_{j=1}^{N} b_{ij} \sin(\theta_i^{(0)} - \theta_j^{(0)}) \,, \text{for } i = 1, ..., N \,. \tag{5}$$

$$\delta\dot{\theta}_i = - \sum_{j=1}^{N} b_{ij} \cos(\theta_i^{(0)} - \theta_j^{(0)})(\delta\theta_i - \delta\theta_j) + \eta_i(t) \,, \text{for } i = 1, ..., N \,. \tag{6}$$

$\eta_i(t)$: Noise inputs.

Robustness of synchronous networks to noise inputs

Near equilibrium dynamics

$$\boldsymbol{\delta\dot{\theta}} = -\mathbb{L}(\{\theta_k^{(0)}\})\boldsymbol{\delta\theta} + \boldsymbol{\eta}(t)\,, \text{for } i = 1, ..., N\,. \tag{7}$$

$\mathbb{L}(\{\theta_k^{(0)}\})$: Jacobian, with eigenvalues $\lambda_1 = 0 < \lambda_2 \leq ... \leq \lambda_N$.

Robustness of synchronous networks to noise inputs

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Solution

$$\delta\theta_i(t) = \sum_\alpha e^{-\lambda_\alpha\, t} \int_0^t e^{\lambda_\alpha\, t'} \sum_j \eta_j(t') u_{\alpha,j} \mathrm{d}t'\, u_{\alpha,i}\,. \tag{8}$$

Robustness of synchronous networks to noise inputs

Uncorrelated white noise

$$\langle \eta_i(t)\eta_j(t')\rangle = \eta_0^2\,\tau_0\,\delta_{ij}\,\delta(t-t') \tag{9}$$

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Variance at node *i*

$$\langle \delta\theta_i^2\rangle = \frac{\eta_0^2\,\tau_0}{2}\sum_\alpha \frac{u_{\alpha,i}^2}{\lambda_\alpha} \tag{10}$$

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Average variance in the network

$$N^{-1}\sum_i \langle \delta\theta_i^2\rangle = \frac{\eta_0^2\,\tau_0}{2N}\sum_\alpha \frac{1}{\lambda_\alpha} \tag{11}$$

Robustness of synchronous networks to noise inputs

Time-correlated noise

$$\langle \eta_i(t)\eta_j(t')\rangle = \eta_0^2\,\delta_{ij}\,e^{-|t-t'|/\tau_0} \tag{12}$$

Robustness of synchronous networks to noise inputs

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Variance at node i for $\tau_0 \gg \lambda_\alpha^{-1}$

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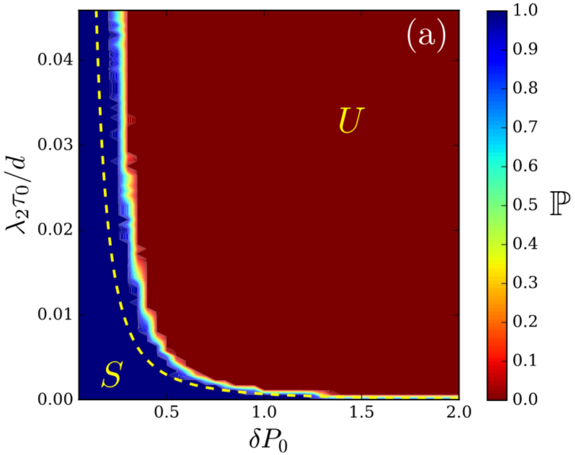
Average variance in the network

$$N^{-1}\sum_i \langle \delta\theta_i^2\rangle = \frac{\eta_0^2}{N}\sum_\alpha \frac{1}{\lambda_\alpha^2} \tag{14}$$

Robustness of synchronous networks to noise inputs

Escape from the initial basin of attraction



MT, Delabays, Jacquod, Phys. Rev. E **99** (6), 062213 (2019)

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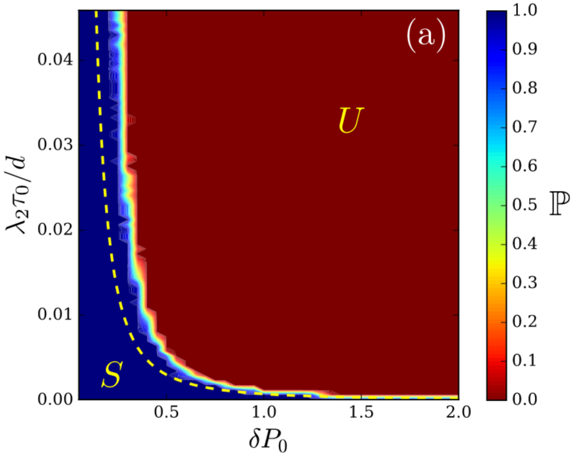
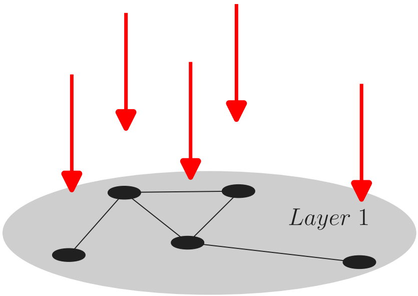


It can be even worst!

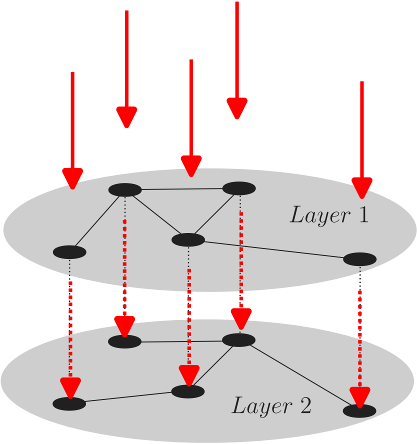
MT, Delabays, Jacquod, Phys. Rev. E **99** (6), 062213 (2019)

Layered Networks



MT, J. Phys. Complex. **3**, 03LT01 (2022)
MT, Chaos **32**(12), 121102, *fast track* (2022)

Layered Networks



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Layered Networks: Multistability

Layered Kuramoto oscillators:

$$\begin{aligned}\dot{\phi}_i &= \omega_i^{(1)} - \sum_{j=1}^{N} b_{ij}^{(1)} \sin(\phi_i - \phi_j) + \eta_i \quad i = 1, \dots N\,, \\ \dot{\theta}_i &= \omega_i^{(2)} - \sum_{j=1}^{N} b_{ij}^{(2)} \sin(\theta_i - \theta_j) + f_i(\{\phi_k\}, \{\theta_k\}) \quad i = 1, \dots N, ,\end{aligned} \tag{15}$$

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$$\mathbb{L}_{ij}^{(1)}(\{\phi_i^{(0)}\}) = \begin{cases} -b_{ij}^{(1)} \cos(\phi_i^{(0)} - \phi_j^{(0)})\,, & i \neq j\,,\\ \sum_k b_{ik}^{(1)} \cos(\phi_i^{(0)} - \phi_k^{(0)})\,, & i = j\,,\end{cases}$$

$$\mathbb{L}_{ij}^{(2)}(\{\theta_i^{(0)}\}) = \begin{cases} -b_{ij}^{(2)} \cos(\theta_i^{(0)} - \theta_j^{(0)})\,, & i \neq j\,,\\ \sum_k b_{ik}^{(2)} \cos(\theta_i^{(0)} - \theta_k^{(0)})\,, & i = j\,,\end{cases}$$

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Two sets of time-scales:

$$\lambda_1^{(1)} = 0 < \lambda_2^{(1)} \leq \ldots \leq \lambda_N^{(1)} \tag{16}$$
$$\lambda_1^{(2)} = 0 < \lambda_2^{(2)} \leq \ldots \leq \lambda_N^{(2)} \tag{17}$$

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\end{aligned} \tag{15}$$

Uncorrelated white noise: $\langle \eta_i(t)\eta_j(t')\rangle = \eta_0^2\, \delta_{ij}\, \delta(t - t')$

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Simplest choice: $f_i(\{\phi_k\}, \{\theta_k\}) = d(\phi_i - N^{-1} \sum_j \phi_j)$

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Noise in the 2nd layer: $\langle \phi_i(t)\phi_j(t') \rangle = \frac{\eta_0^2}{2} \sum_\alpha \frac{u_{\alpha,i}^{(1)} u_{\alpha,j}^{(1)}}{\lambda_\alpha^{(1)}} e^{-\lambda_\alpha^{(1)}|t-t'|}$.

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Layered Networks

Analytical treatment:

$$\phi_i(t) = \sum_\alpha e^{-\lambda_\alpha^{(1)} t} \int_0^t e^{\lambda_\alpha^{(1)} t'} \sum_j \eta_j \, u_{\alpha,j}^{(1)} \, \mathrm{d}t' \, u_{\alpha,i}^{(1)} \,, \tag{16}$$

$$\theta_i(t) = \sum_\alpha e^{-\lambda_\alpha^{(2)} t} \int_0^t e^{\lambda_\alpha^{(2)} t'} \sum_j \phi_j \, u_{\alpha,j}^{(2)} \, \mathrm{d}t' \, u_{\alpha,i}^{(2)} \,. \tag{17}$$

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Layer 1:

$$\langle \phi_i^2 \rangle = \frac{\eta_0^2}{2} \sum_\alpha \frac{{u_{\alpha,i}^{(1)}}^2}{\lambda_\alpha^{(1)}} \,, \tag{18}$$

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Layer 2:

$$\langle \theta_i^2 \rangle = \frac{\eta_0^2}{2} \sum_{\alpha,\beta,\gamma} \sum_{k,l} \frac{u_{\gamma,k}^{(1)} u_{\gamma,l}^{(1)} u_{\alpha,k}^{(2)} u_{\beta,l}^{(2)} [2\lambda_\gamma^{(1)} + \lambda_\alpha^{(2)} + \lambda_\beta^{(2)}]}{\lambda_\gamma^{(1)} (\lambda_\alpha^{(2)} + \lambda_\beta^{(2)})(\lambda_\gamma^{(1)} + \lambda_\alpha^{(2)})(\lambda_\gamma^{(1)} + \lambda_\beta^{(2)})} u_{\alpha,i}^{(2)} u_{\beta,i}^{(2)} \,. \tag{19}$$

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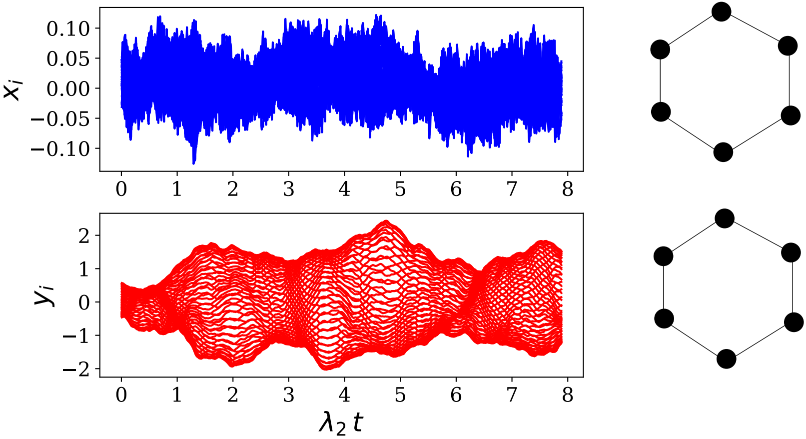
Layer 1:

$$\langle \phi_i^2 \rangle = \frac{\eta_0^2}{2} \sum_\alpha \frac{{u_{\alpha,i}^{(1)}}^2}{\lambda_\alpha^{(1)}} \,, \quad (18)$$

Layer 2: Same networks

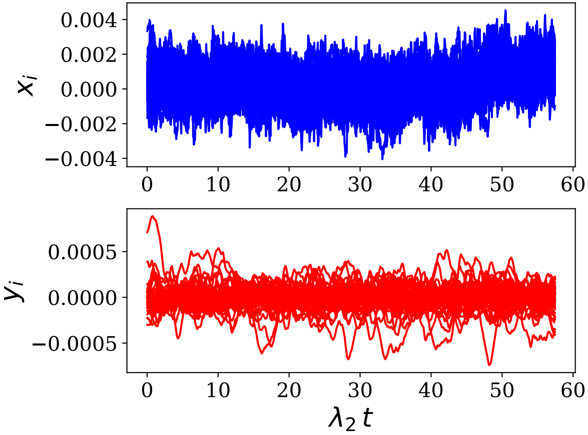
$$\langle \theta_i^2 \rangle = \frac{\eta_0^2}{4} \sum_\alpha \frac{u_{\alpha,i}^2}{\lambda_\alpha^3} \,. \quad (19)$$

Layered Networks: Amplification



MT, J. Phys. Complex. **3**, 03LT01 (2022)

Layered Networks: Amplification



MT, J. Phys. Complex. **3**, 03LT01 (2022)

Conclusion and future work

So far

- Linearization around an equilibrium point $\rightarrow$ structure of the coupling network:
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  - System-specific correlations: MT J. Phys: Complex. **3**(3), 03LT01 (2022); MT Chaos **32** (12) (2022).
  - Heterogeneous and correlated noise: J.Hindes, I.B.Schwartz arxiv:2308.13434 (2023).

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Future work

- Weak coupling and mobile oscillators.

Recent Advances in Learning and Data-Driven Modeling of Complex Systems

Satellite session of CCS 2023, Salvador, Bahia, Brazil.
October 18 or 19, 2023

Nowadays, more and more data are collected and accessible about systems as diverse as interactions on social media networks, financial flows on stock exchange cryptocurrencies markets, frequency variations on both power transmission and distribution grids to name but a few. This increasing amount of system measurements enables first, a more precise modelling of dynamical systems. Indeed, using a large amount of data, one may verify if the established mathematical models are correct and detailed enough to capture the dynamics, or if some refinement of the theory is necessary. Second, from the data, system-specific parameters may be learned, which is a crucial task in order to predict the system's future behavior. These two missions are both overlapping and complementary, and represent timely challenges in various fields of physics, mathematics, engineering, and computer science.
The main objective of our symposium is to gather experts in learning and data-driven modelling of dynamical systems coming from very different fields as different scientific communities are approaching these questions from different perspectives.