How to pick the right mathematical model? Using optimal control for closed-loop model discrimination.

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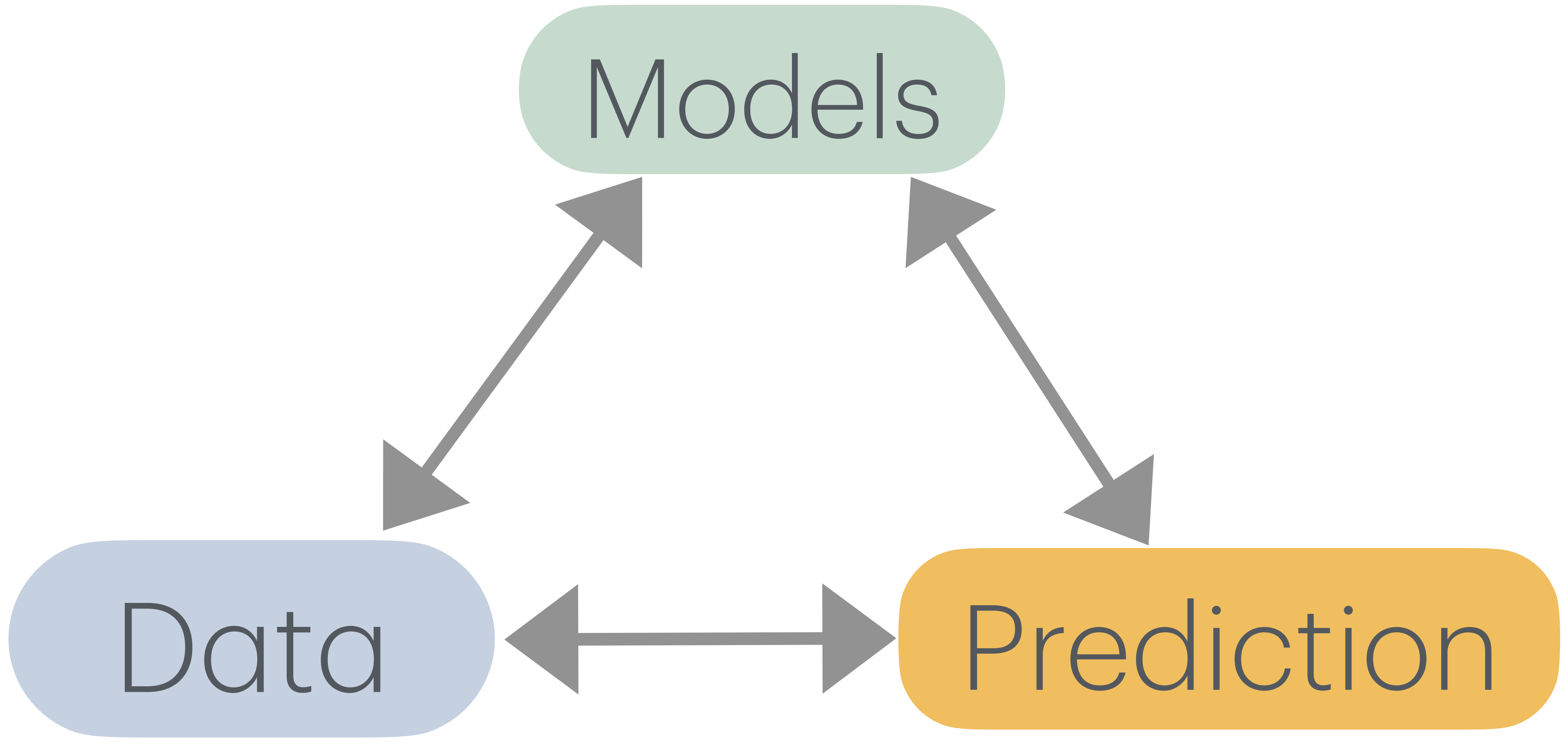

Dynamics Seminar, Exeter — 19/3/2026

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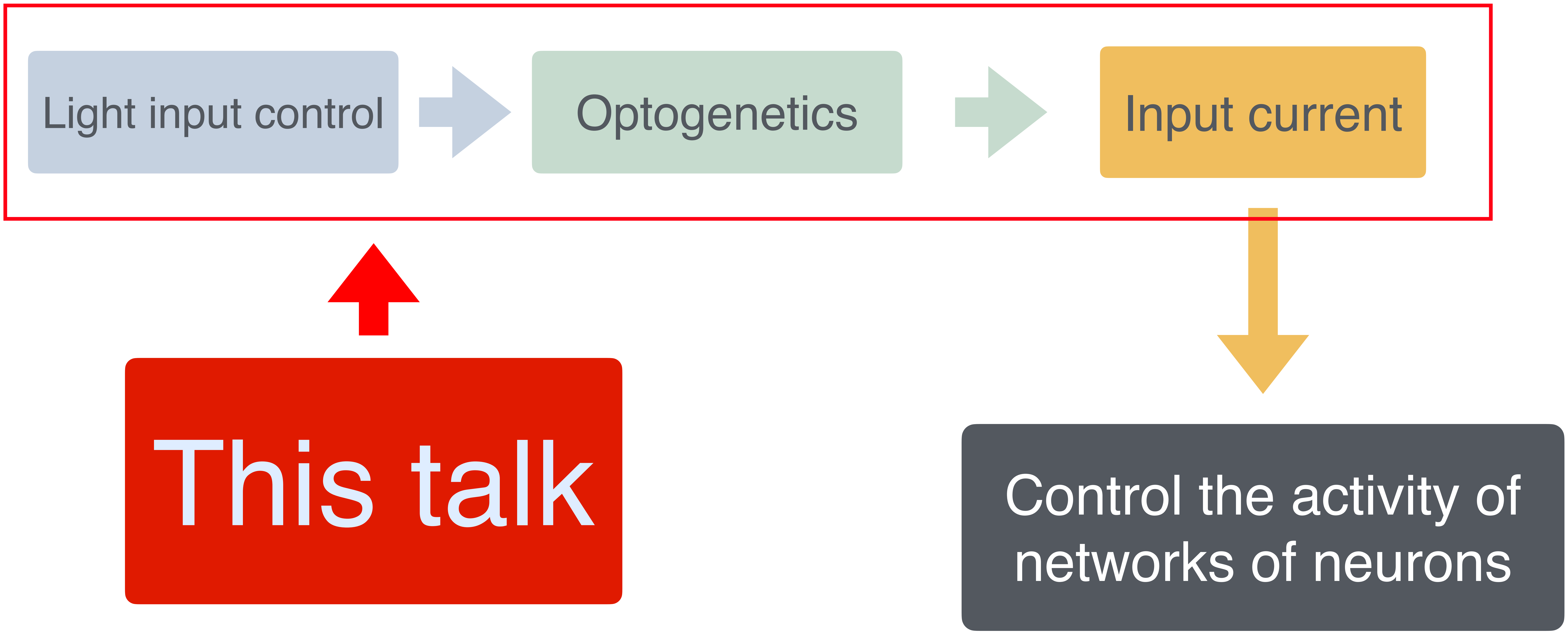
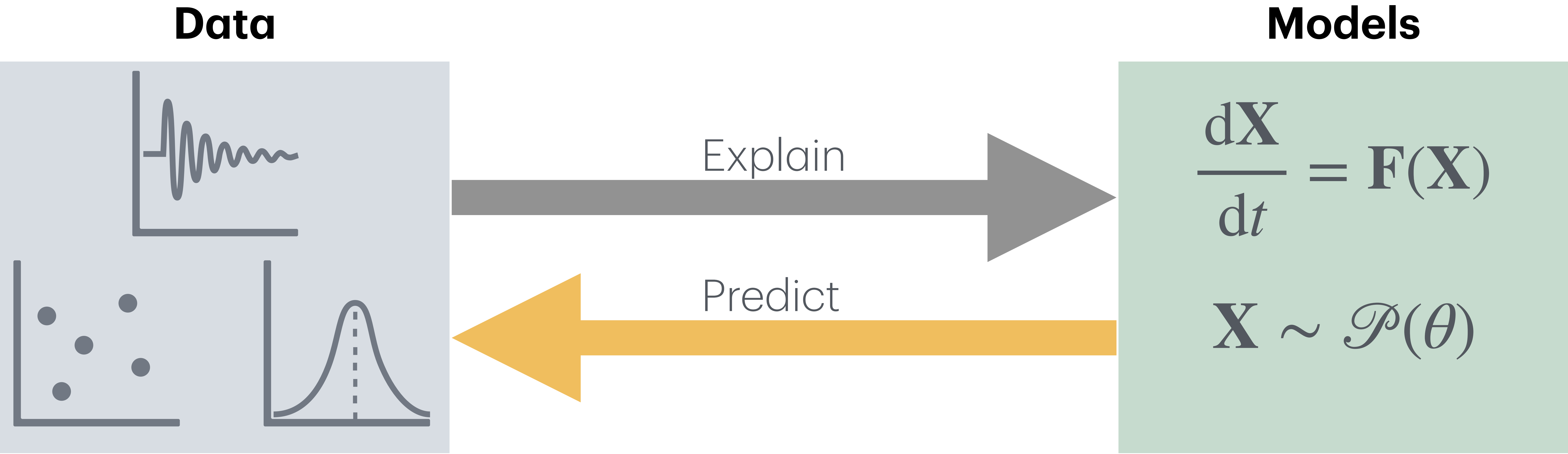
Preprint arXiv:2603.00709

Why model discrimination?



Data — Models

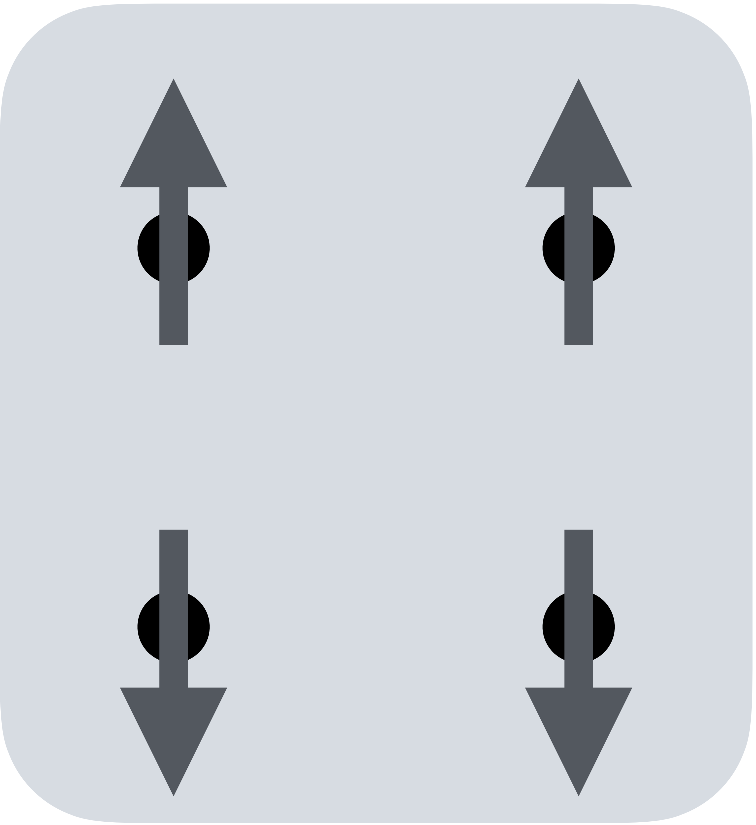


- Measurements from an experiment
- Observation of a process

- Differential equations
- Probabilistic models

More than one model…

Observed system



Average magnetization

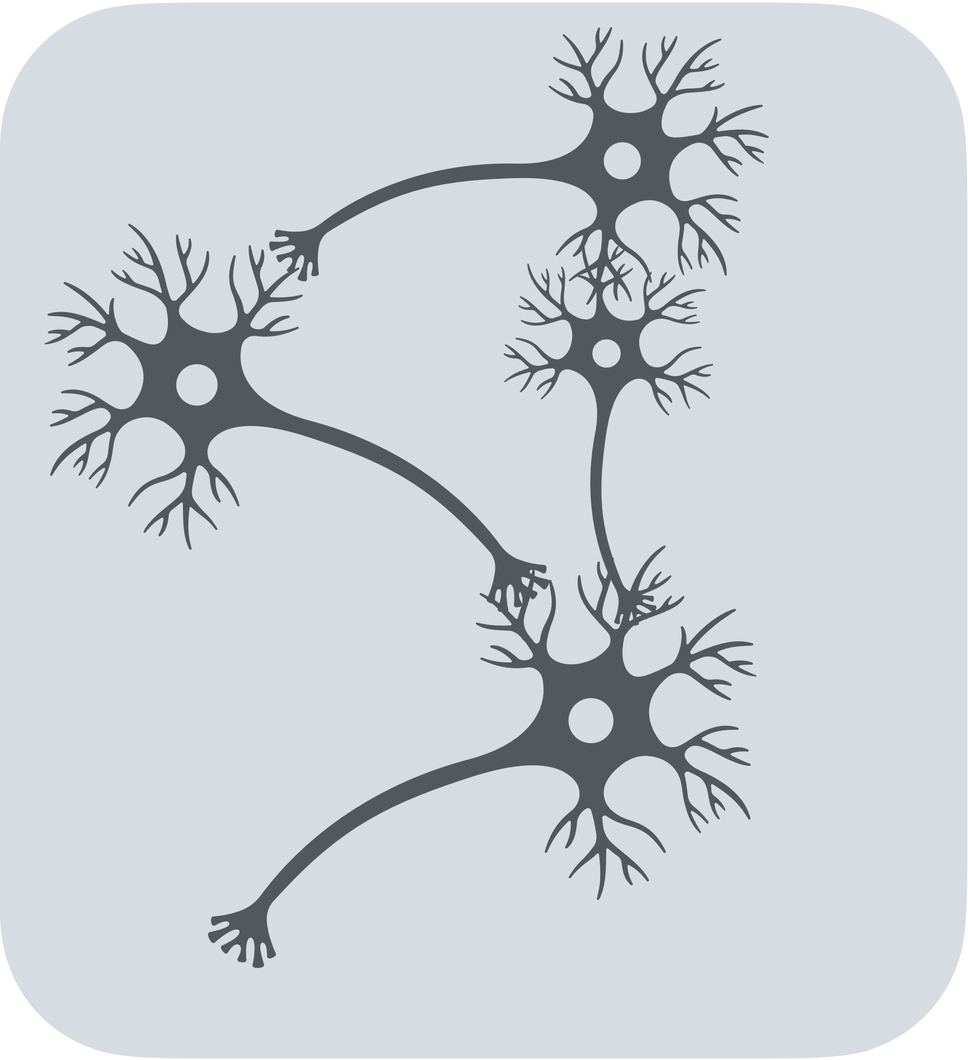
Models

$$\mathscr{H} = \sum_{<i,j>} J_{ij}\sigma_i\sigma_j$$

$$\mathscr{H} = \sum_{<i,j>} T_{ij} c^{\dagger}_{\sigma} c_{\sigma} + \alpha \sum_{i} n^{\dagger}_{\sigma} n_{\sigma}$$

More than one model…

Observed system



Spike times

Models

$$\tau \dot{V}_i = V_i^2 + \eta_i + \sum_{j=1}^{N} A_{ij}(V_i - V_j) + I_i^{\text{ext}}(t)\,,$$

$$V_i(t) = V_{\text{th}} \rightarrow V_i(t) = -\,V_{\text{th}}$$

$$\dot{V}_i = V_i - \frac{V_i^3}{3} - w + \alpha I_i^{\text{ext}}(t)\,,$$

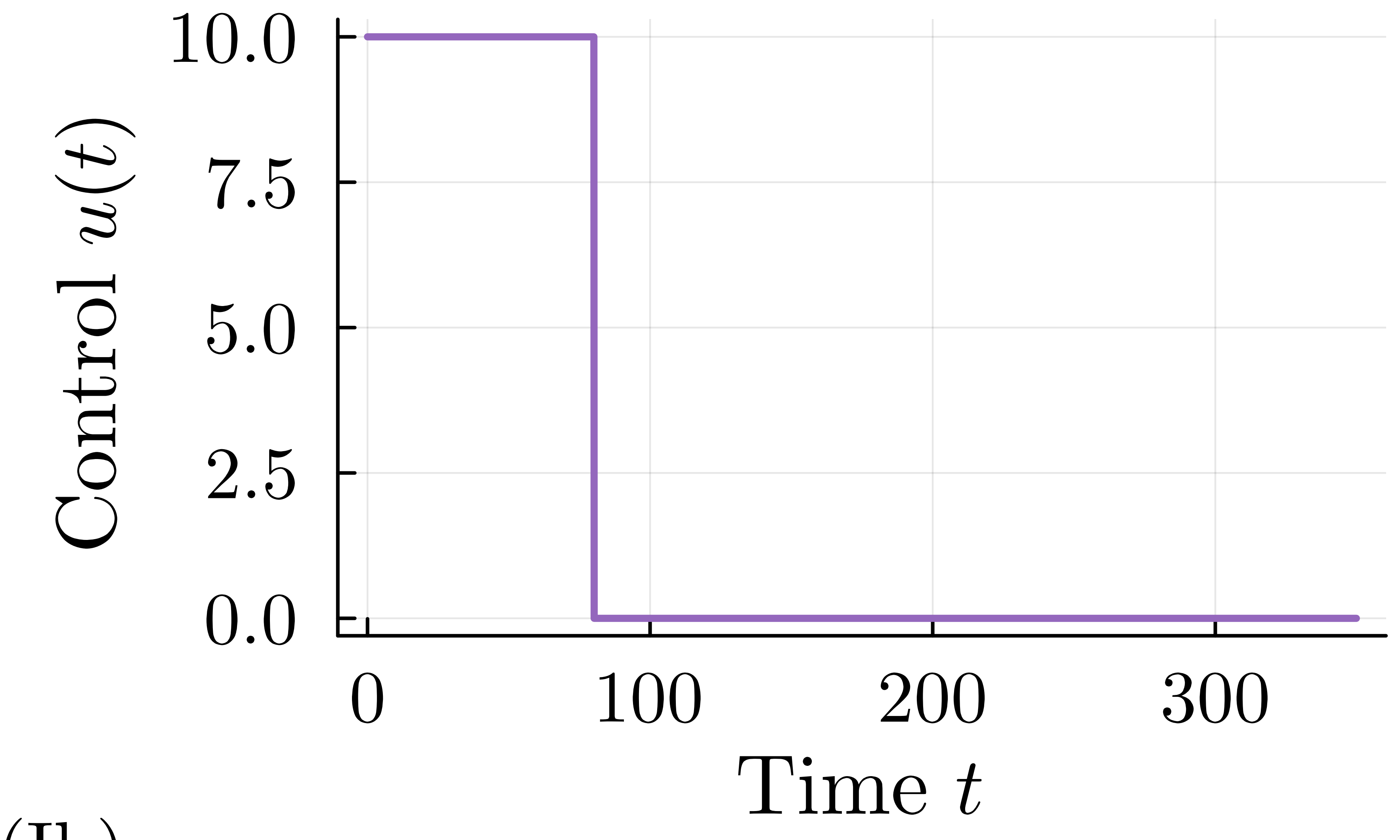
$$\tau \dot{w}_i = V_i + a - b w_i$$

Candidate models

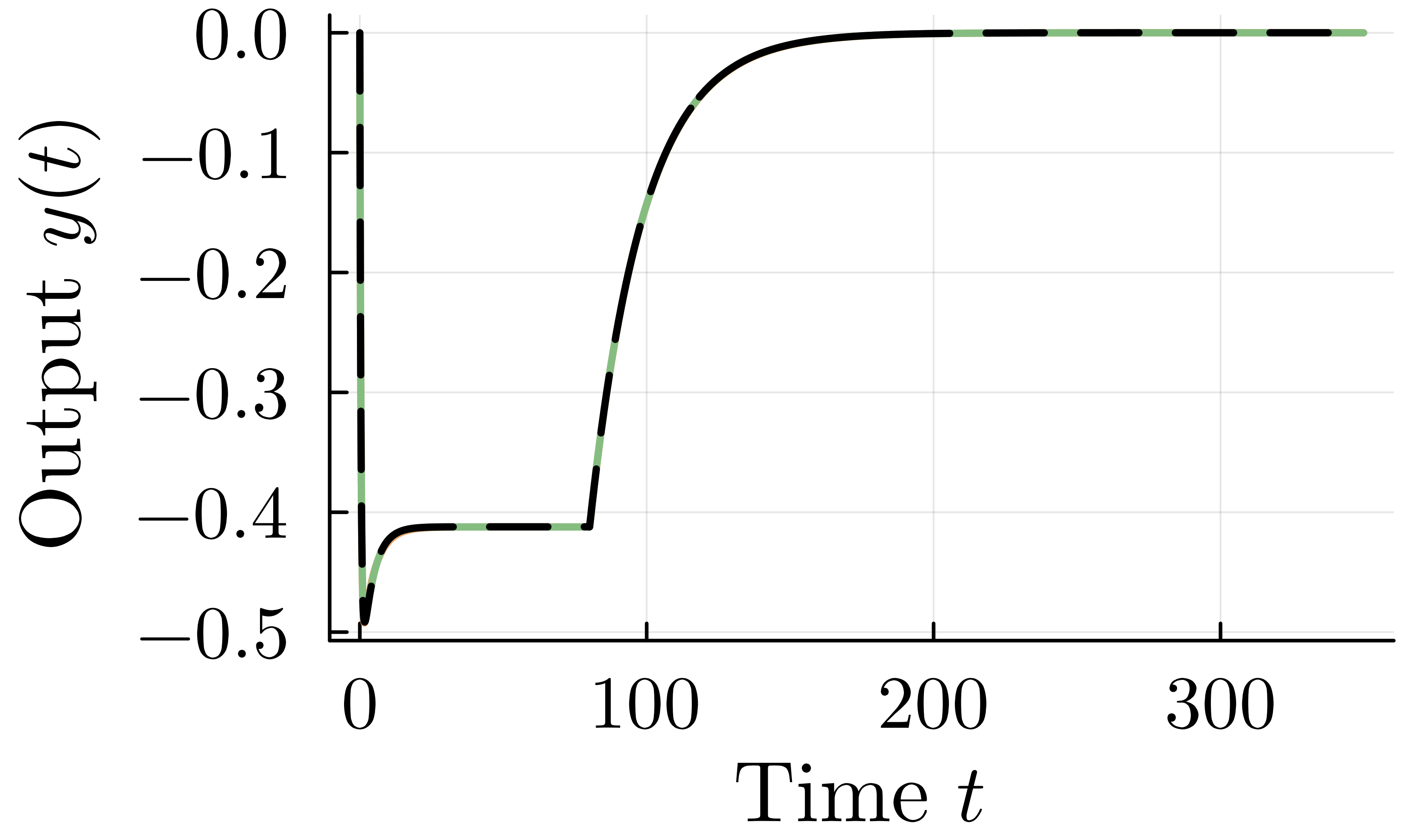
$$\dot{\boldsymbol{X}}_k = \boldsymbol{F}_k\left(\boldsymbol{X}_k, u(t)\,; \boldsymbol{\Theta}_k\right),$$

$$\boldsymbol{Y}_k = \boldsymbol{O}_k\left(\boldsymbol{X}_k;\, \boldsymbol{\Theta}_k\right), \qquad k = 1,\ldots N\,.$$

(Ia)



(Ib)

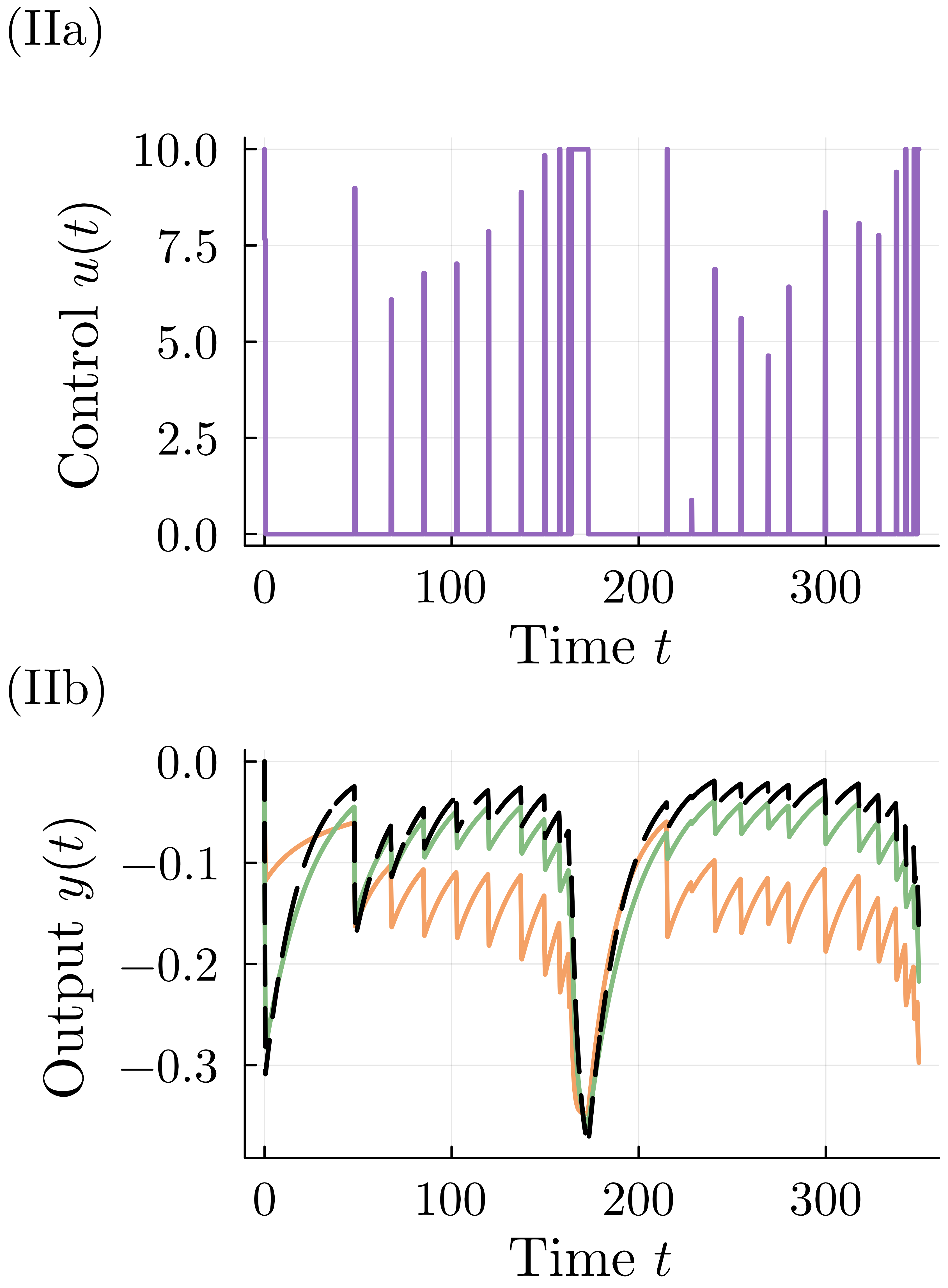


Candidate models

$$\dot{\boldsymbol{X}}_k = \boldsymbol{F}_k\left(\boldsymbol{X}_k, u(t)\,; \boldsymbol{\Theta}_k\right)\,,$$

$$\boldsymbol{Y}_k = \boldsymbol{O}_k\left(\boldsymbol{X}_k;\, \boldsymbol{\Theta}_k\right)\,, \qquad k = 1,\ldots N\,.$$

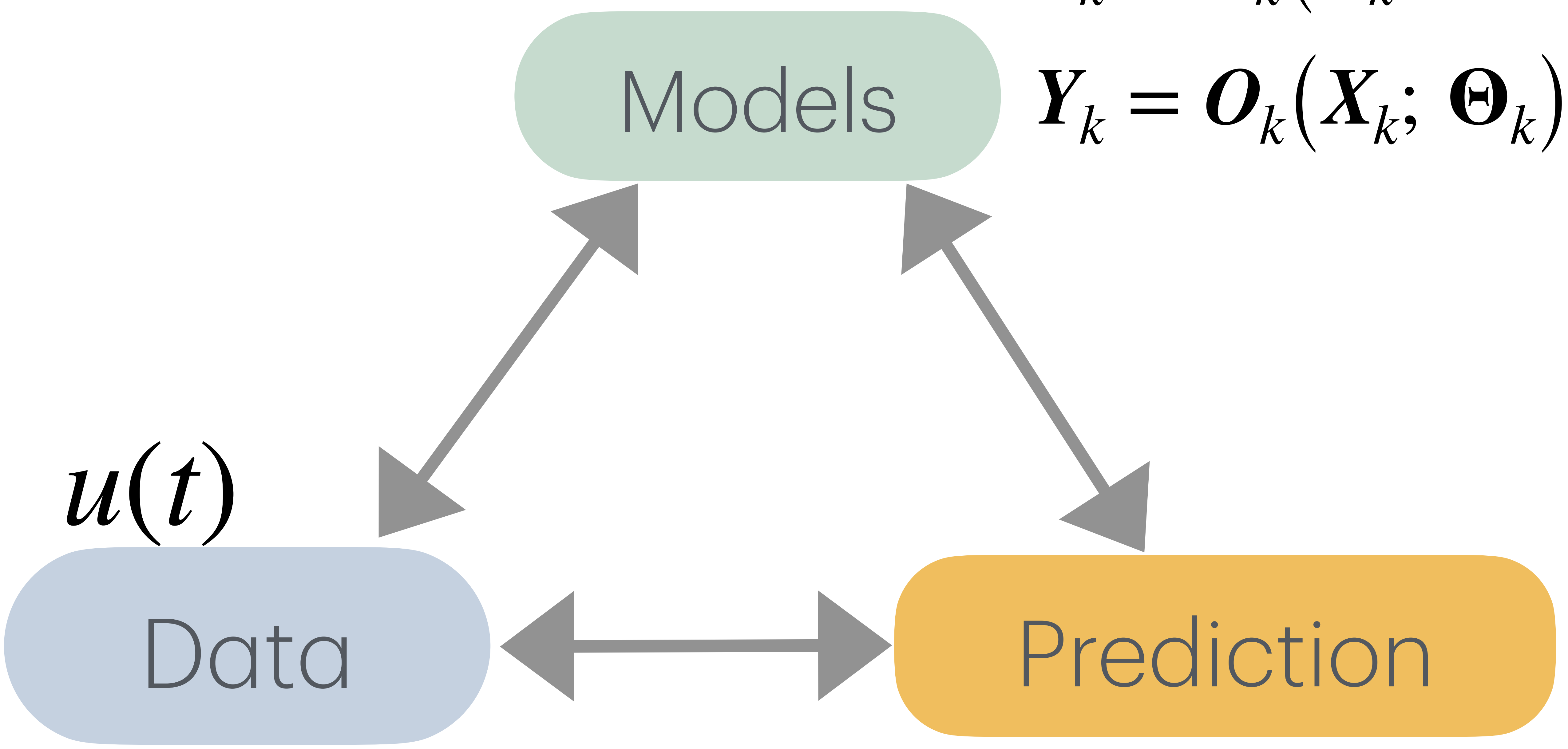
Optimal control



The objective

- For a set of candidate models, find control inputs that allow to choose the right model.

$$\dot{\boldsymbol{X}}_k = \boldsymbol{F}_k\left(\boldsymbol{X}_k, u(t)\,; \boldsymbol{\Theta}_k\right),$$
$$\boldsymbol{Y}_k = \boldsymbol{O}_k\left(\boldsymbol{X}_k; \boldsymbol{\Theta}_k\right), \qquad k = 1,\ldots N.$$



Step 0

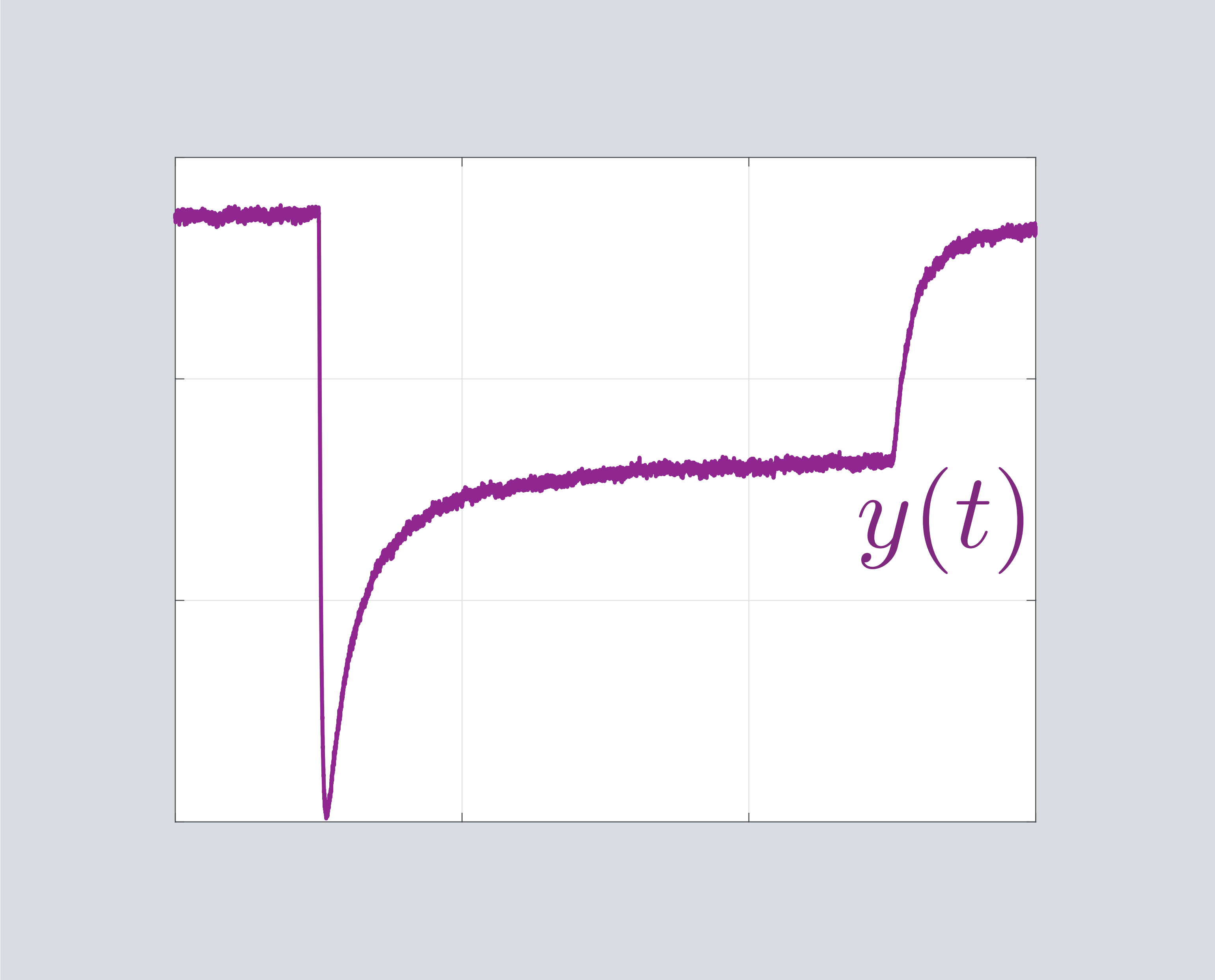
Find some candidate models!

$$\dot{\boldsymbol{X}}_k = \boldsymbol{F}_k\left(\boldsymbol{X}_k, u(t)\,; \boldsymbol{\Theta}_k\right)\,,$$

$$\boldsymbol{Y}_k = \boldsymbol{O}_k\left(\boldsymbol{X}_k; \boldsymbol{\Theta}_k\right)\,, \qquad k = 1{,}2.$$

Step 1 → Step 2

Collect some data



Parameter fitting

$$\dot{\boldsymbol{X}}_k = \boldsymbol{F}_k\left(\boldsymbol{X}_k, u(t)\,; \boldsymbol{\Theta}_k\right),$$

$$\boldsymbol{Y}_k = \boldsymbol{O}_k\left(\boldsymbol{X}_k;\ \boldsymbol{\Theta}_k\right), \qquad k = 1,2.$$

Fitting using a loss function

$$\mathscr{L}(\boldsymbol{\Theta}_k; k) = \int_0^T D\Big(\mathbf{Z}(t), \mathbf{Y}\big(\boldsymbol{X}_k(t)\big)\Big)\,\mathrm{d}t$$

$$D(\boldsymbol{x}, \boldsymbol{y}) = \|\boldsymbol{x} - \boldsymbol{y}\|_2^2$$

Step 3

Optimal Control

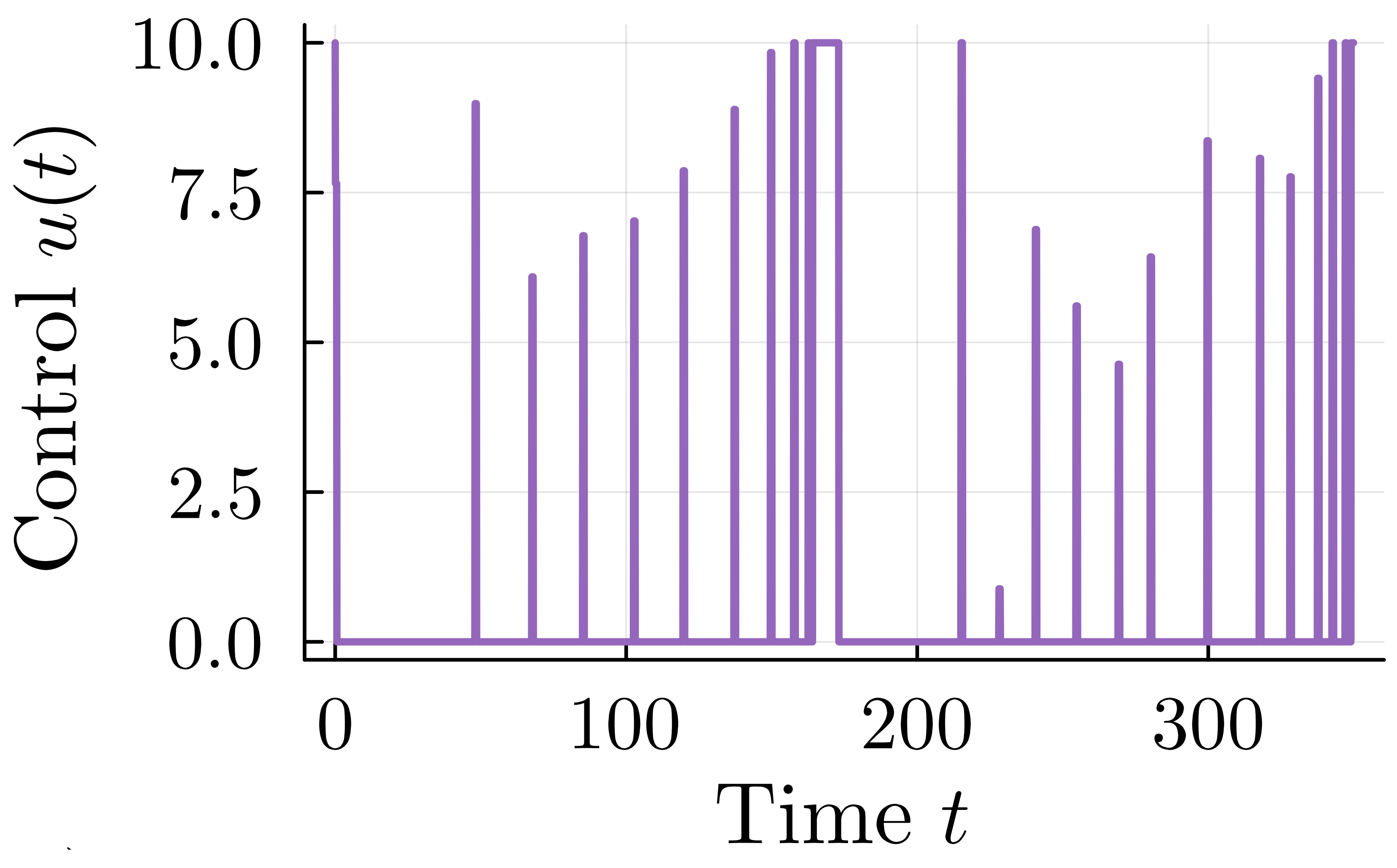
$$\mathcal{J}\big(u(t)\big) := \int_0^T \Big[D\Big(\boldsymbol{Y}_1\big(X_1(u)\big), \boldsymbol{Y}_2\big(\boldsymbol{X}_2(u)\big)\Big) - C(u)\Big]\,\mathrm{d}t$$

$$u^* = \mathrm{argmax}_u\,\mathcal{J}(u)$$

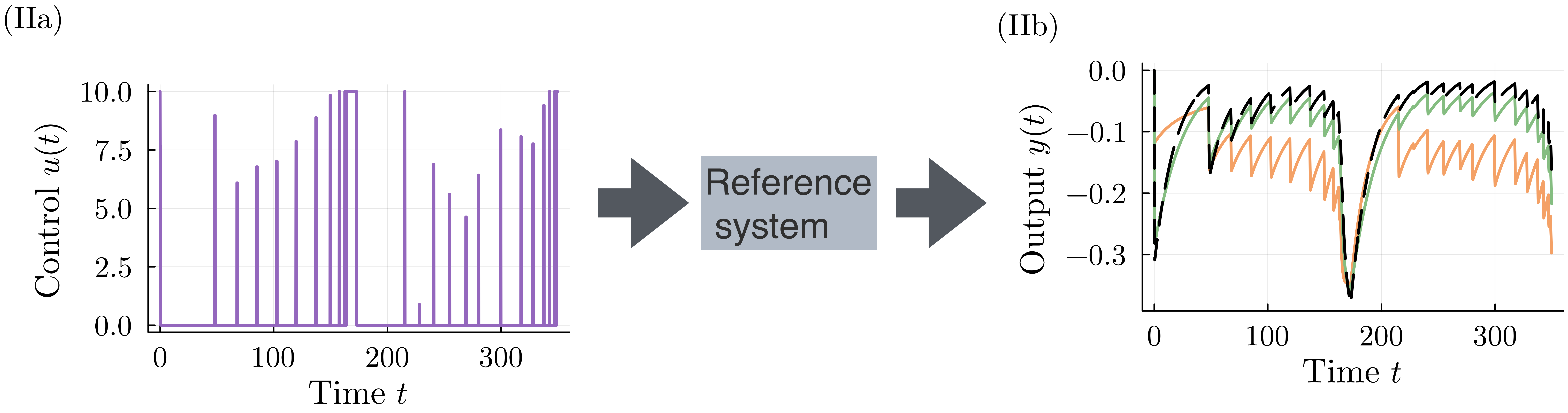
Constraint on past control signals!

Efficiently solved using Ipopt.

(IIa)

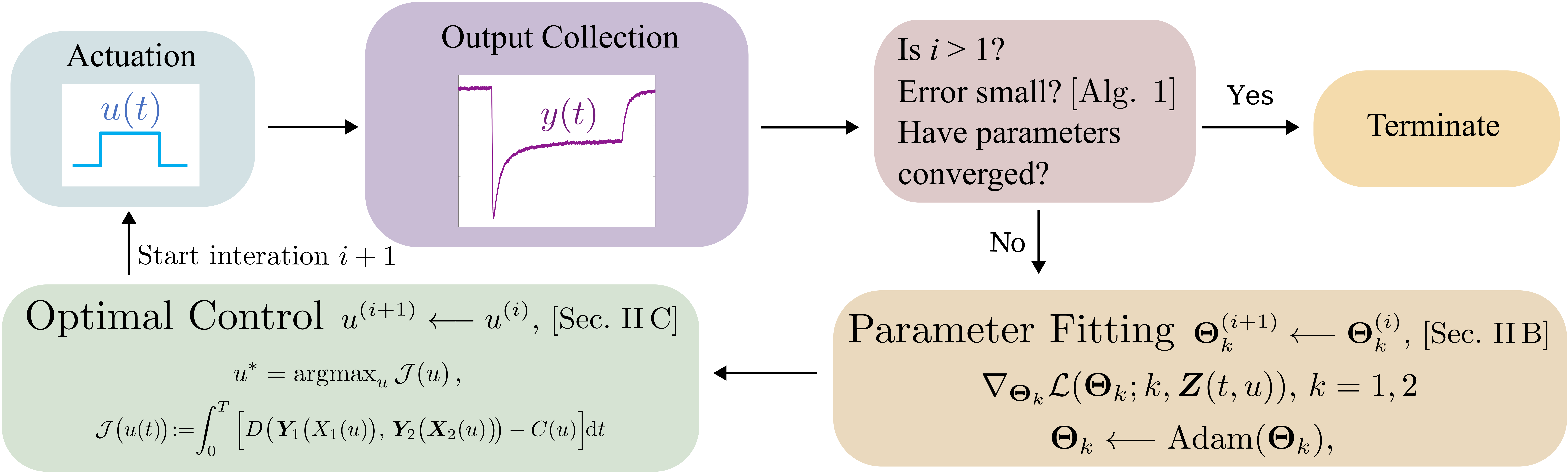


Step 4 Step 0

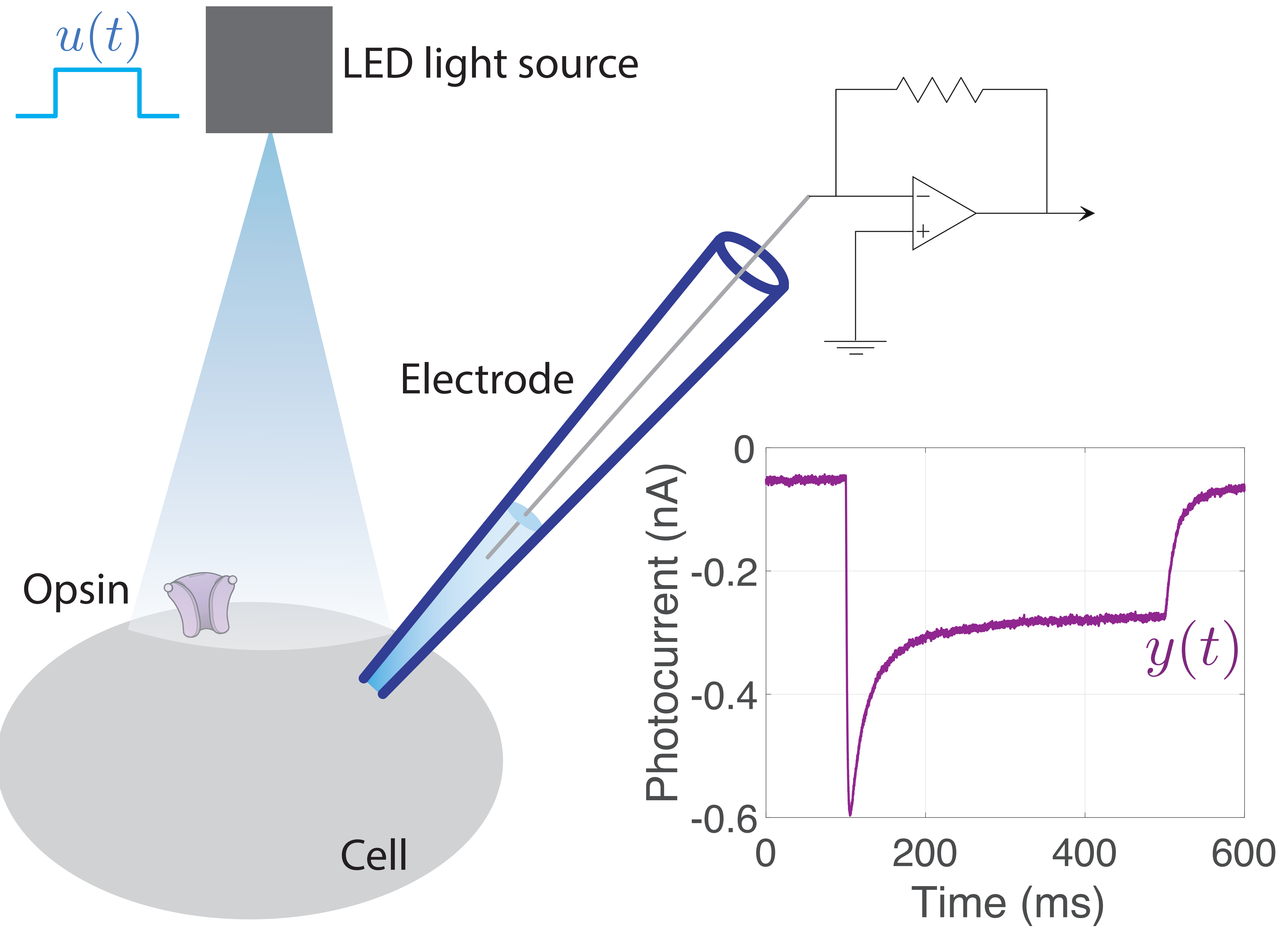


(IIa)
Control $u(t)$
10.0
7.5
5.0
2.5
0.0
0
100
200
300
Time t
Reference system
(IIb)
Output $y(t)$
0.0
−0.1
−0.2
−0.3
0
100
200
300
Time t

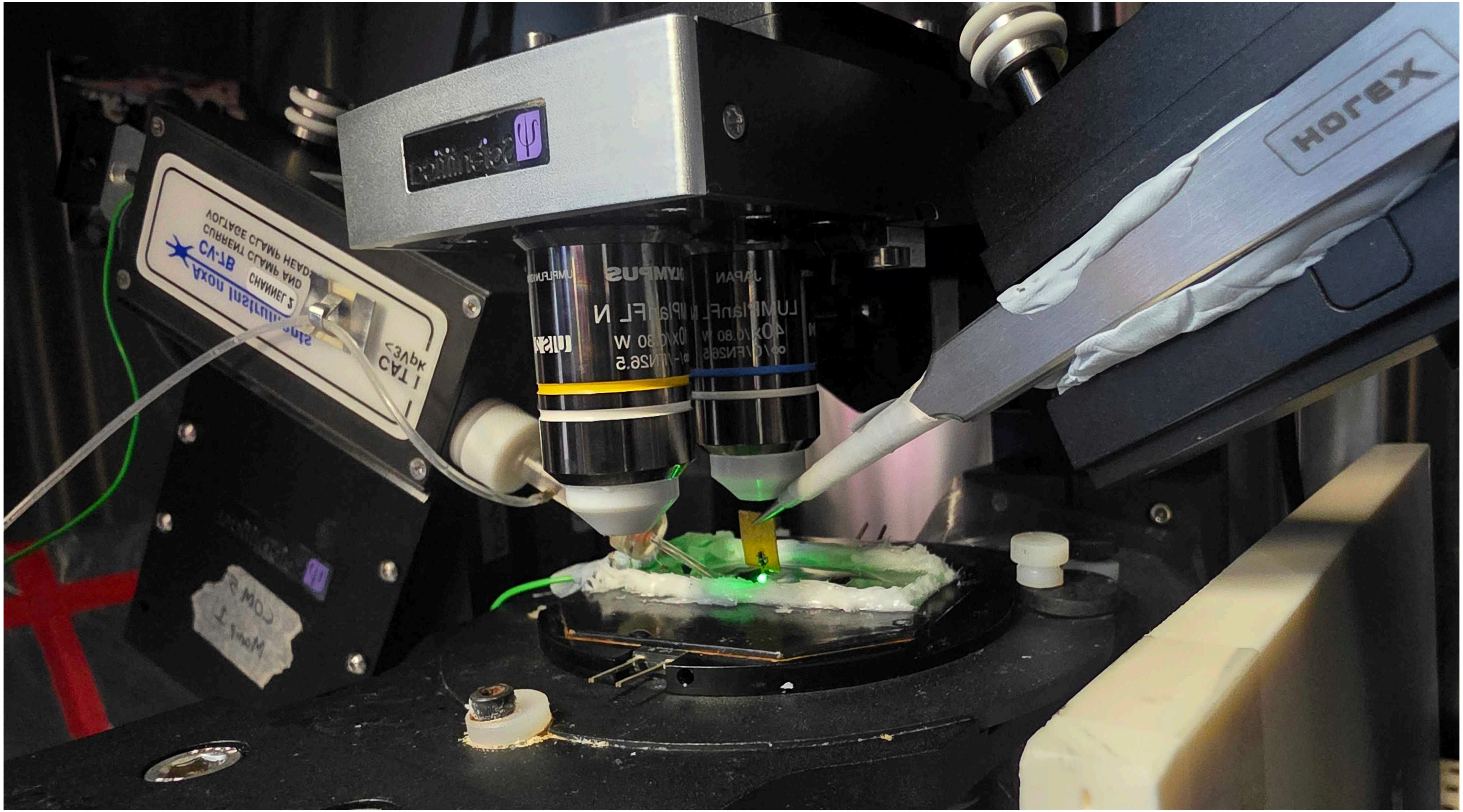
Method Workflow



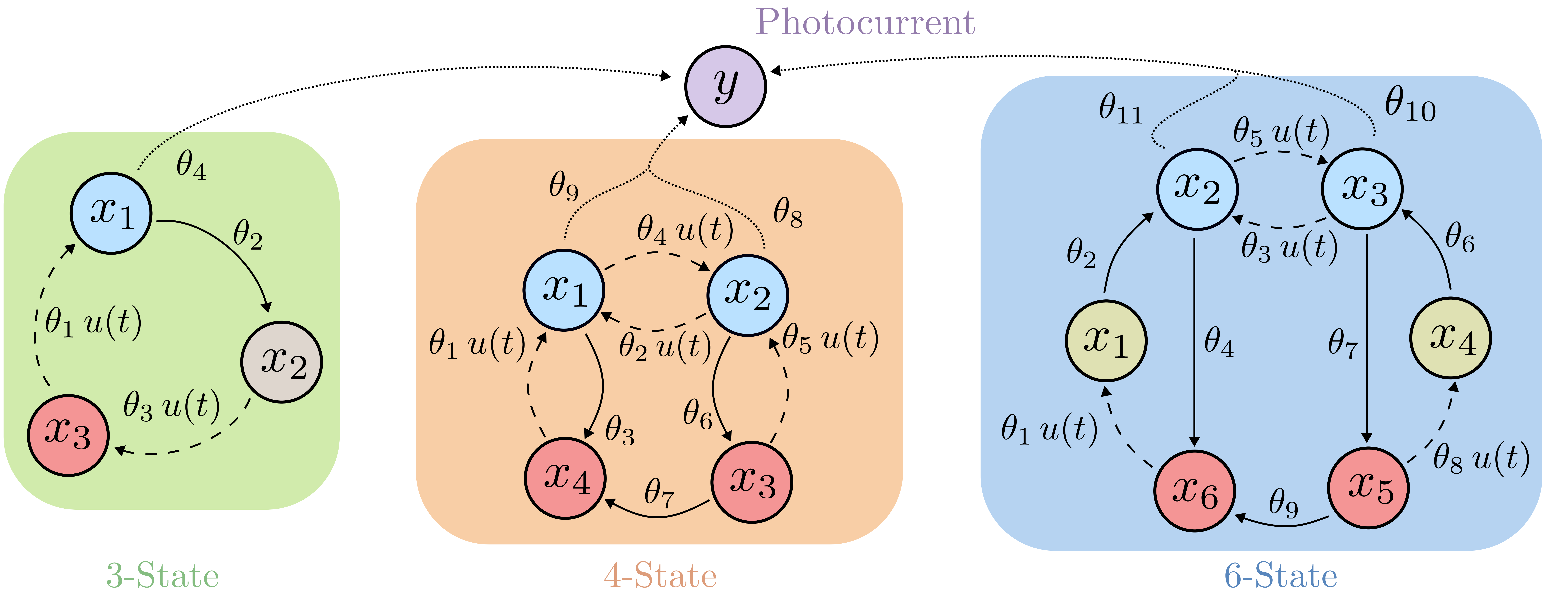
Electrophysiology experiment



Experiment



Opsin models



Evans et al. Frontiers '16

Opsin models

3-state model

$$\dot{x}_1 = \theta_1 u(t)\, x_3 - \theta_2 x_1\,,$$
$$\dot{x}_2 = \theta_2 x_1 - \theta_3 u(t)\, x_2\,,$$
$$x_3 = 1 - x_1 - x_2\,,$$
$$y = \theta_4 x_1\,.$$

4-state model

$$\dot{x}_1 = \theta_1 u(t)\, x_4 + \theta_2 u(t)\, x_2 - \left[\theta_3 + \theta_4 u(t)\right] x_1\,,$$
$$\dot{x}_2 = \theta_5 u(t)\, x_3 + \theta_4 u(t)\, x_1 - \left[\theta_6 + \theta_2 u(t)\right] x_2\,,$$
$$\dot{x}_3 = \theta_6\, x_2 - \left[\theta_7 + \theta_5 u(t)\right] x_3\,,$$
$$x_4 = 1 - x_1 - x_2 - x_3\,,$$
$$y = \theta_9\, x_1 + \theta_8\, x_2\,.$$

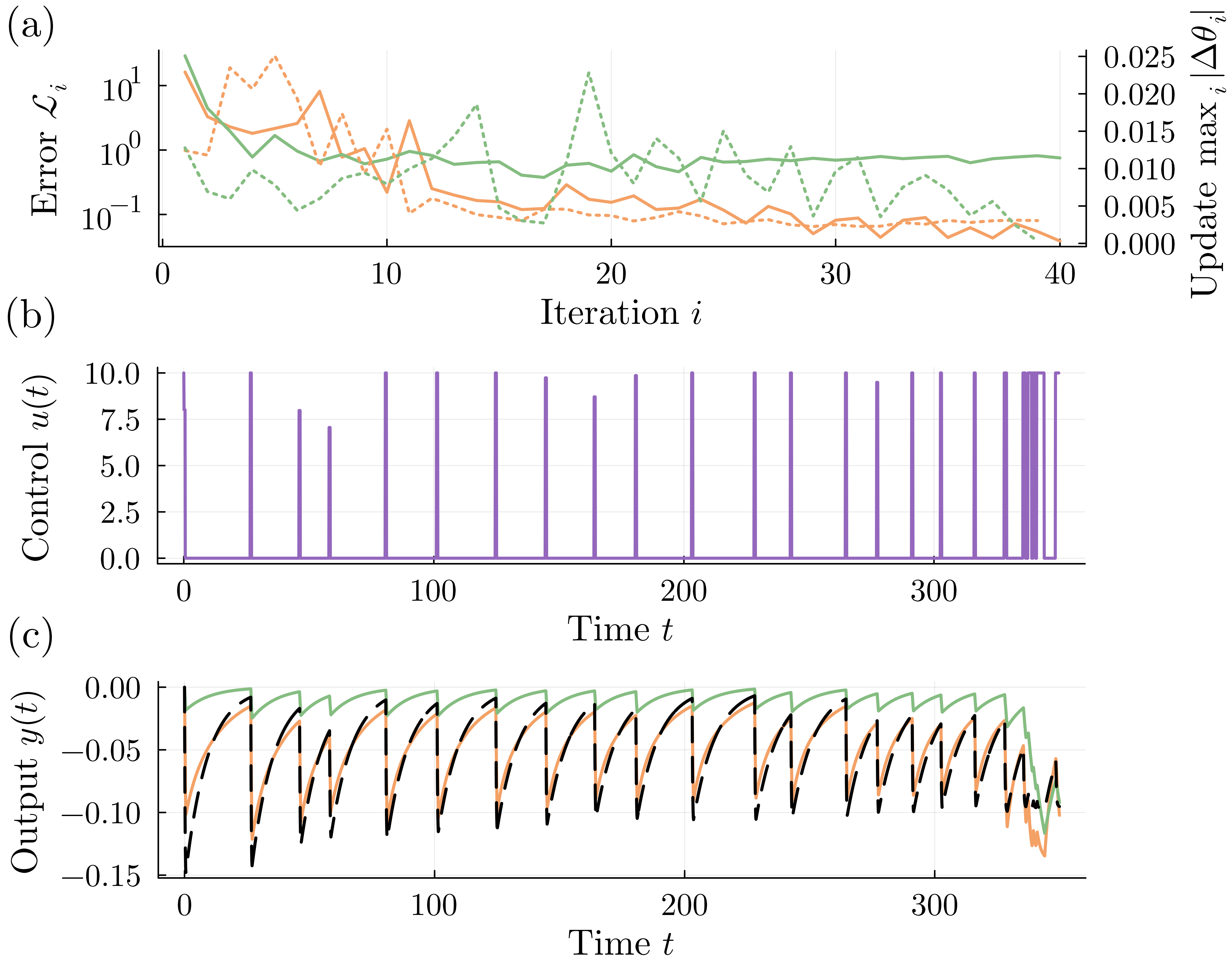
6-state model

$$\dot{x}_1 = \theta_1 u(t)\, x_6 - \theta_2 x_1\,,$$
$$\dot{x}_2 = \theta_2 x_1 + \theta_3 u(t)\, x_3 - \left[\theta_4 + \theta_5 u(t)\right] x_2\,,$$
$$\dot{x}_3 = \theta_6 x_4 + \theta_5 u(t)\, x_2 - \left[\theta_7 + \theta_3 u(t)\right] x_3\,,$$
$$\dot{x}_4 = \theta_8 u(t)\, x_5 - \theta_6 x_4\,,$$
$$\dot{x}_5 = \theta_7\, x_3 - \left[\theta_9 + \theta_8 u(t)\right] x_5\,,$$
$$x_6 = 1 - x_1 - x_2 - x_3 - x_4 - x_5\,,$$
$$y = \theta_{11}\, x_2 + \theta_{10}\, x_3\,.$$

Numerical Simulations

Candidate models:
3-state vs. 4-state

Reference system:
4-state



Numerical Simulations

Ref: 3-State	4-State	6-State
3-State	0.090(0.182) | 1.334(0.181)	0.432(0.736) | 3.393(3.465)
4-State	×	0.978(0.492) | 1.281(0.491)

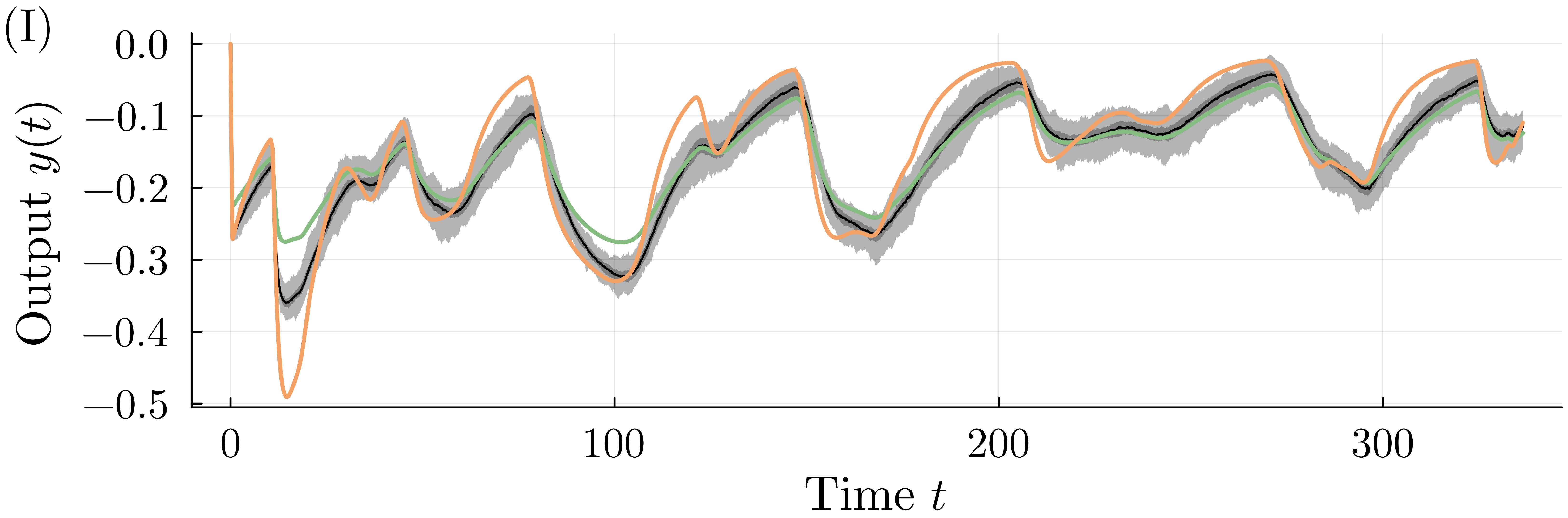
Ref: 4-State	4-State	6-State
3-State	0.320(0.176) | 0.228(0.118)	0.469(0.169) | 0.284(0.094)
4-State	×	0.230(0.193) | 1.044(2.463)

Ref: 6-State	4-State	6-State
3-State	0.571(0.338) | 0.226(0.132)	0.473(0.255) | 0.279(0.142)
4-State	×	0.725(1.287) | 0.718(1.414)

Numerical Simulations

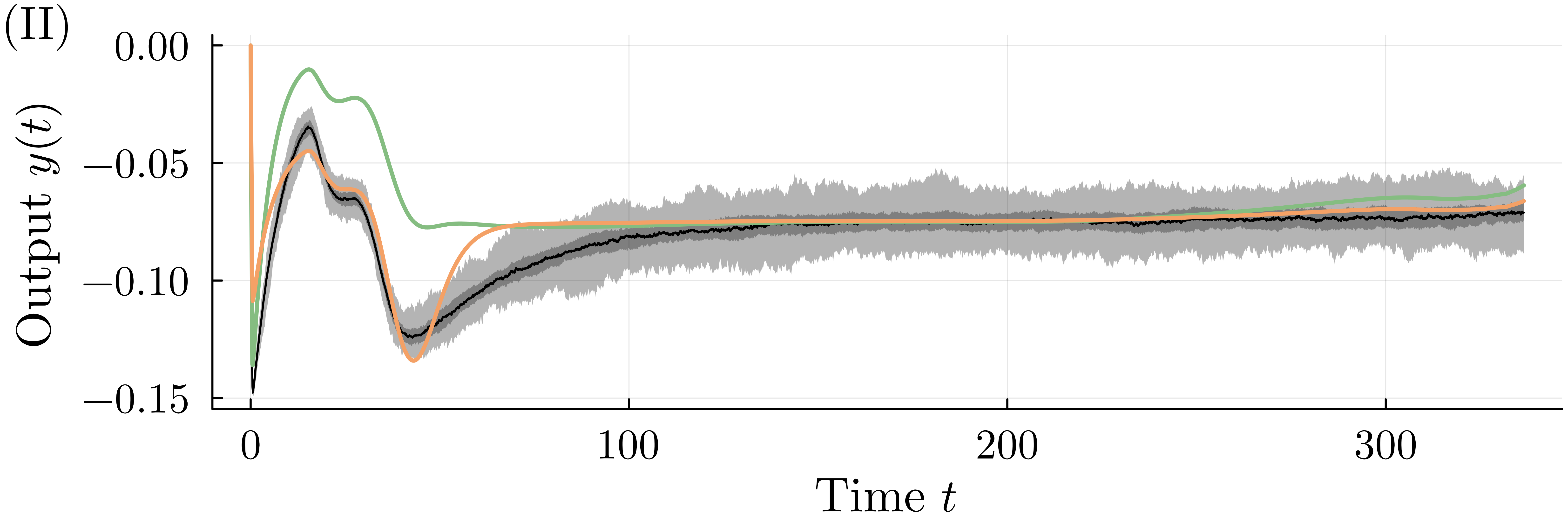
Candidate models:
3-state vs. 4-state

Reference system:
3-state

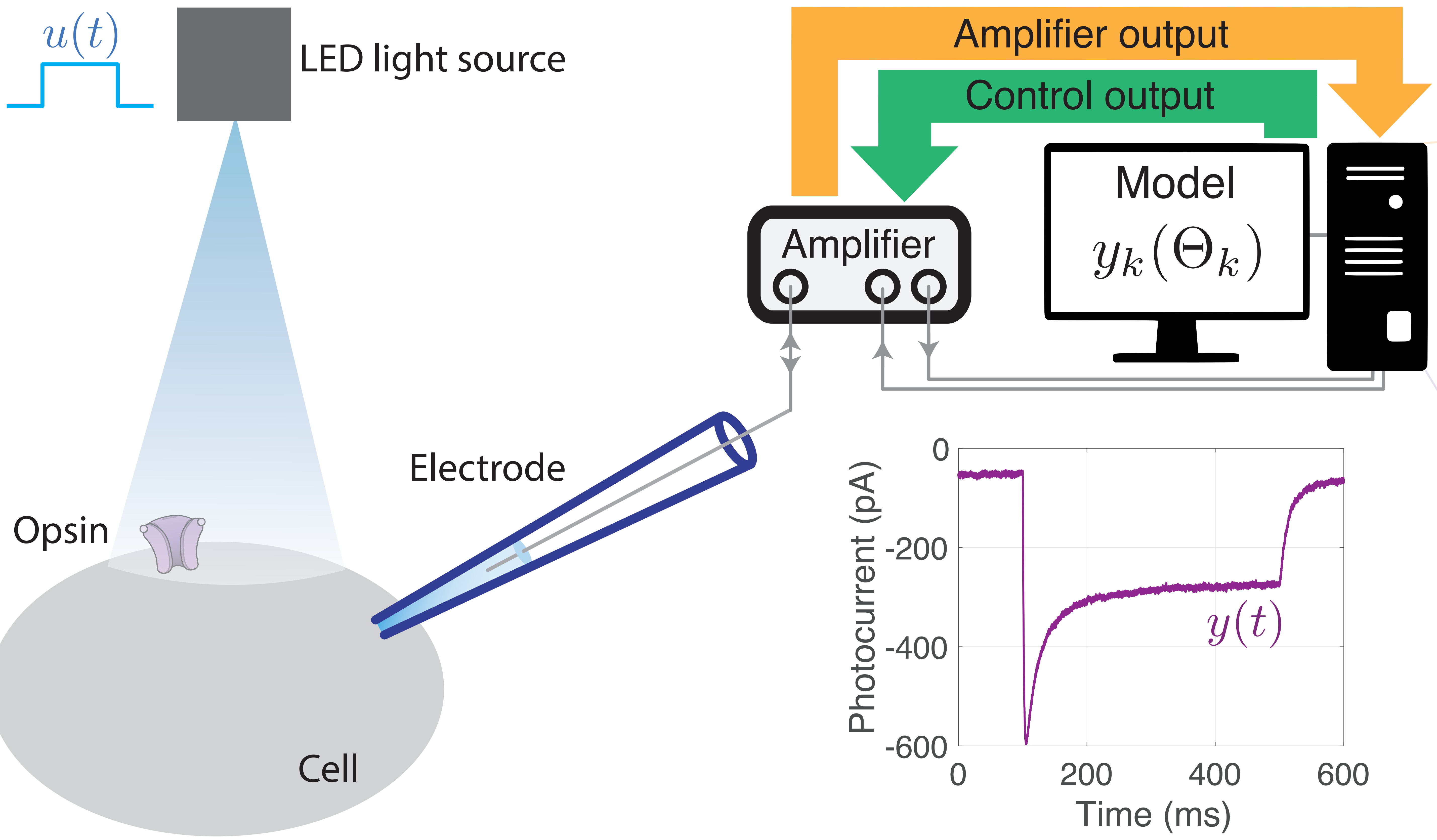


Candidate models:
3-state vs. 4-state

Reference system:
4-state



Computer — Experiment



Symphony is an OOP Matlab-based app for electrophysiology recording

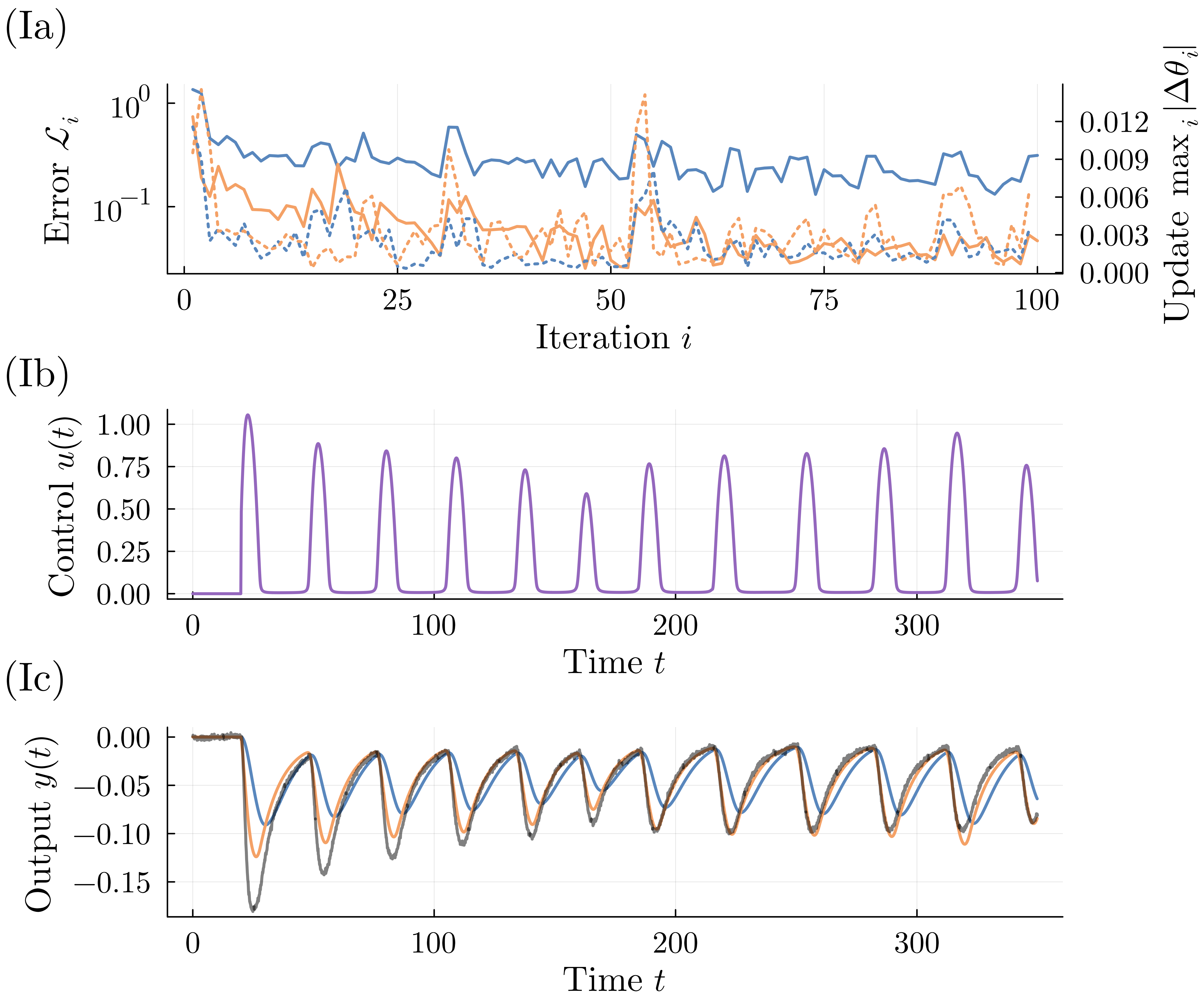
~10s

Sockets.jl to open *tcp server* to connect to following each data collection step

~5s

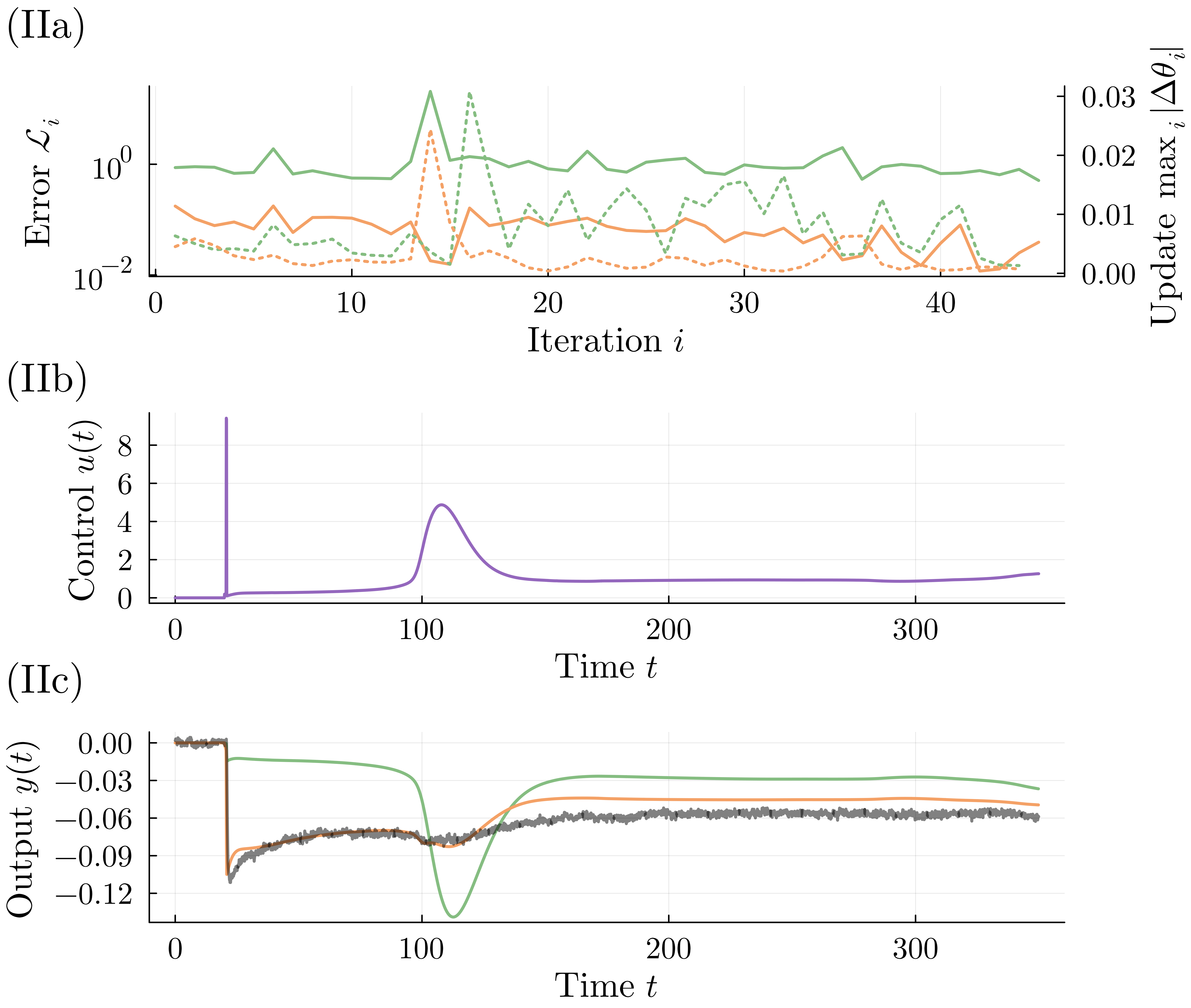
Experiment

Candidate models:
6-state vs. 4-state



Experiment

Candidate models:
3-state vs. 4-state



Conclusion

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- Closed-loop algorithm for model selection.
- Allowed to select the most accurate opsin model.
- Exploration of the phase space based on heuristics.

Future work

- Build digital twins.
- Leverage digital twin to optimize the control inputs.
- Apply to other systems.

Preprint arXiv:2603.00709

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