Primary control effort and noise propagation in high-voltage power grids

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High-voltage power grids of the future

- More fluctuations from renewable energy sources.
- Less inertia at generator nodes.

Goals

- ➢ Evaluate importance of damping and inertia parameters.
- ➢ Assess the propagation of fluctuations coming from renewable energy sources.

Swing equations in the lossless line approximation

Voltage phase dynamics is given by

$$m_i\,\dot{\omega}_i + d_i\,\omega_i = P_i - \sum_j b_{ij}\sin(\theta_i-\theta_j)\,,\quad i\in\text{Generators}\,, \tag{1}$$

$$d_i\,\omega_i = P_i - \sum_j b_{ij}\sin(\theta_i-\theta_j)\,,\quad i\in\text{Loads}\,. \tag{2}$$

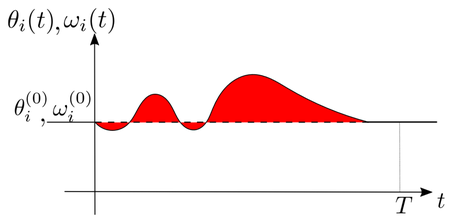
b_{ij} : line capacity.
m_i : inertia.
d_i : damping.
$\omega_i=\dot{\theta}_i$.

Assumption:

$\gamma^{-1}=m_i/d_i\,,\ \forall i$

Assessment of the transient dynamics

Quadratic performance metrics: $\mathcal{H}_2$ norms → Quantify the amplitude of the transient response following a disturbance.



Primary control effort

$$\mathcal{P}(T) = \lim_{T\to\infty} T^{-1}\int_0^T (\boldsymbol{\omega}^\top-\overline{\boldsymbol{\omega}}^\top)D(\boldsymbol{\omega}-\overline{\boldsymbol{\omega}})\,\mathrm{d}t$$

Time-correlated noisy disturbance

$$\langle\delta P_i\rangle=0\,,\ \langle\delta P_i(t)\delta P_j(t')\rangle=\delta_{ij}\delta P_0^2\exp[-|t-t'|/\tau_0]$$

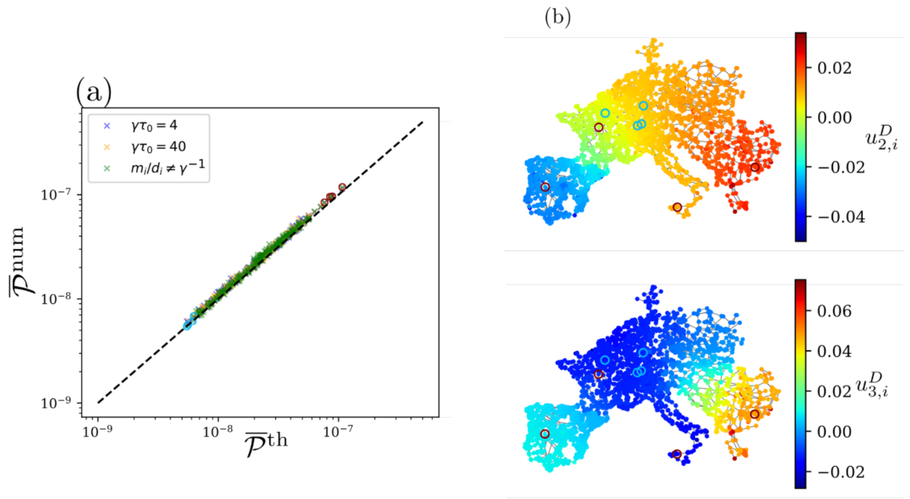
Long correlation time limit

- Intrinsic network time-scales < few seconds
- Renewable energy sources correlation time > few seconds

$$\overline{\mathcal{P}^\infty}=\tau_0^{-1}\sum_{\alpha\geq 2}\frac{\sum_{i\in N_n}\delta P_{0i}^2 {u_{\alpha,i}^D}^2 d_i^{-1}}{\lambda_\alpha^D}$$

λ_α^D :eigenvalues of $D^{-1/2}\mathbb{L}(\{\theta_i^{(0)}\})D^{-1/2}$

PanTaGruEl: model of the European high-voltage transmission network.



Results more general than the initial assumption!

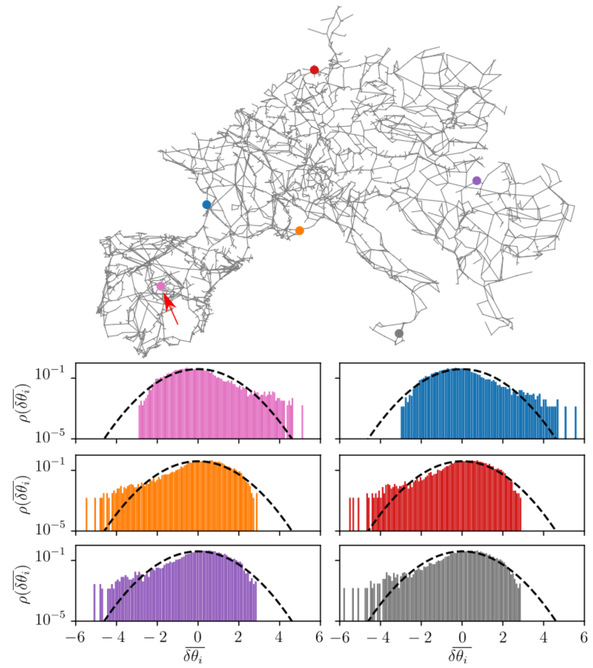
Conclusions

- ✓ Damping seems more important than inertia to reduce primary control effort.
- ✓ Non-Gaussian fluctuations not that of a problem.

Non-Gaussian fluctuations from renewable energy sources

Cumulants for a single source of noise

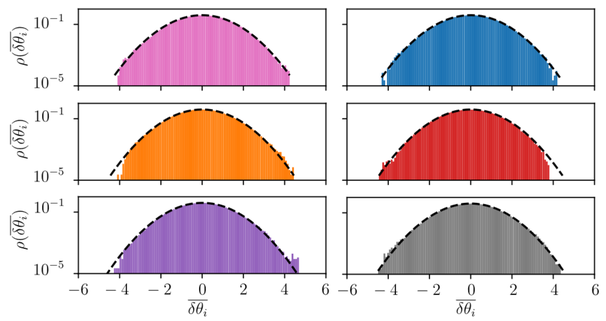
$$\langle\delta\theta_i^p\rangle=\left(\sigma\sum_\alpha\frac{u_{\alpha,i_0}u_{\alpha,i}}{\lambda_\alpha}\right)^p$$



4th moment for M sources of noise

$$\langle\delta\theta_i^4\rangle=\sum_{i_0=1}^M\left(\sigma\sum_\alpha\frac{u_{\alpha,i_0}u_{\alpha,i}}{\lambda_\alpha}\right)^4$$

$$+3\sum_{i_0<j_0}\left(\sigma\sum_\alpha\frac{u_{\alpha,i_0}u_{\alpha,i}}{\lambda_\alpha}\right)^2\left(\sigma\sum_\beta\frac{u_{\beta,j_0}u_{\beta,i}}{\lambda_\beta}\right)^2$$



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References:
- M. Tyloo, P. Jacquod IEEE Control Systems Letters 5 (3), 929-934 (2020)
- M. Tyloo, J. Hindes, P. Jacquod arXiv preprint arXiv:2203.00590 (2022)