# Robustness of synchronous networks

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# Introduction

- Vulnerabilities of networked systems

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- Inference and fault identification

# Synchronization

- Simple experiment: coupled metronomes

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- Other examples:
  - Fireflies flashing in unison
  - People clapping their hands or walking on a bridge
  - Synchronization of the quantum phase of Josephson junctions arrays
  - Synchronization of the phase of the voltages in electric power grids
  - Consensus formation on influence networks
  - Vehicular platoon formation
  - Orchestra

# Synchronization and Music

NATURE COMMUNICATIONS | https://doi.org/10.1038/s41467-020-17540-7 ARTICLE


Shahal et al. Nature communications **11**, 3854 (2020).

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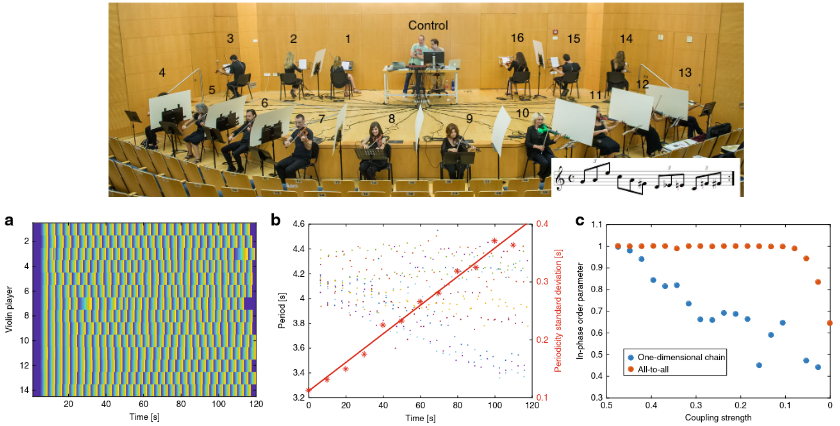


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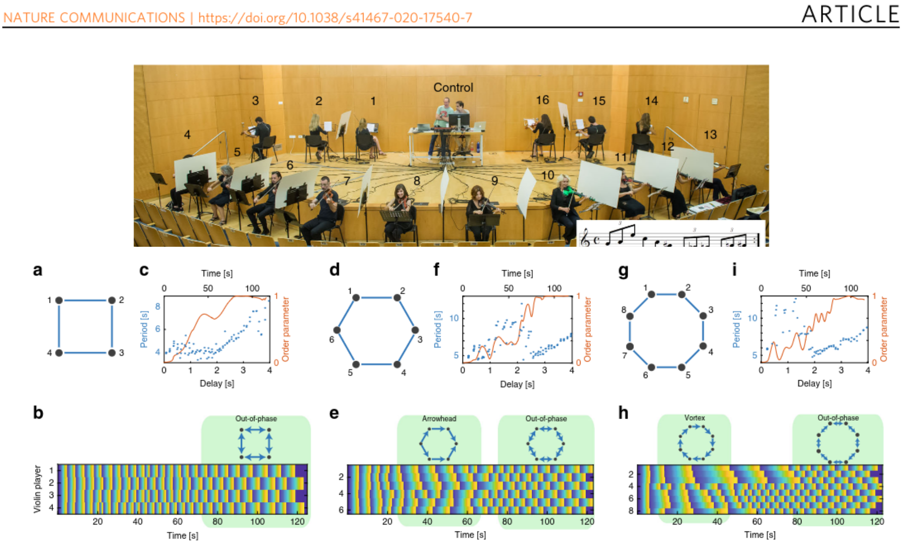


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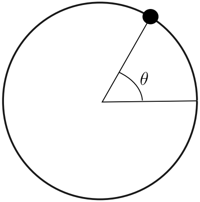
# Modelling Synchronization

Single phase oscillator: $\dot{\theta} = \omega$



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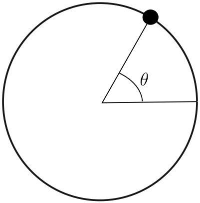
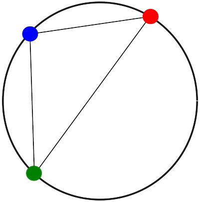



Coupled phase oscillators: $\dot{\theta}_i = \omega_i - \sum_j b_{ij} f(\theta_i - \theta_j)$



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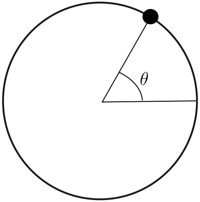


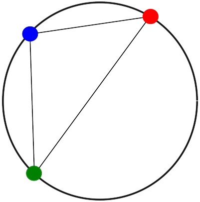

Coupled phase oscillators: $\dot{\theta}_i = \omega_i - \sum_j b_{ij} f(\theta_i - \theta_j)$

**Synchonization**: phase-locked $\dot{\theta}_i(t) = \dot{\theta}_j(t)\,,\ \forall i, j\,.$

**Kuramoto model**

$$\dot{\theta}_i = \omega_i - \frac{K}{N}\sum_{j=1}^{N}\sin(\theta_i - \theta_j)\,, \text{for } i = 1, ..., N\,. \tag{1}$$

$\omega_i$: natural frequencies.

Y. Kuramoto, Lecture Notes in Physics 39, International Symposium on Mathematical Problems in Theoretical Physics (1975).

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**Order parameter**

$$re^{i\,\psi} = N^{-1}\sum_{j=1}^{N} e^{i\,\theta_j} \tag{2}$$

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# Modelling of Synchronization

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$\omega_i$: natural frequencies.

**Order parameter**

$$re^{i\,\psi} = N^{-1}\sum_{j=1}^{N} e^{i\,\theta_j} \quad (2)$$

$\psi$: average phase.

Illustration

$$\dot{\theta}_i = \omega_i - Kr\sin(\psi - \theta_i)\,, \text{for } i = 1, ..., N\,. \quad (3)$$

Y. Kuramoto, Lecture Notes in Physics 39, International Symposium on Mathematical Problems in Theoretical Physics (1975).

**Kuramoto model**

$$\dot{\theta}_i = \omega_i - \sum_{j=1}^{N} b_{ij} \sin(\theta_i - \theta_j)\,, \text{for } i = 1, ..., N\,. \tag{4}$$

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$b_{ij}$: adjacency matrix.

Good survey Dörfler and Bullo, Automatica **50** (6), 1539-1564, (2014)

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**Multistability**

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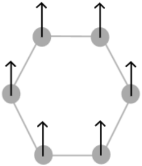

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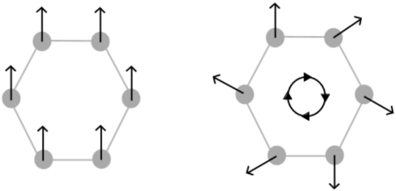

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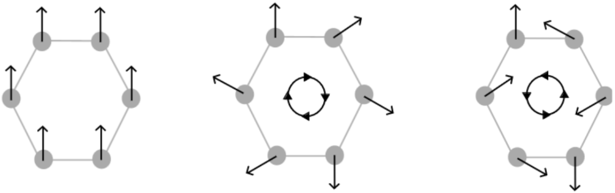

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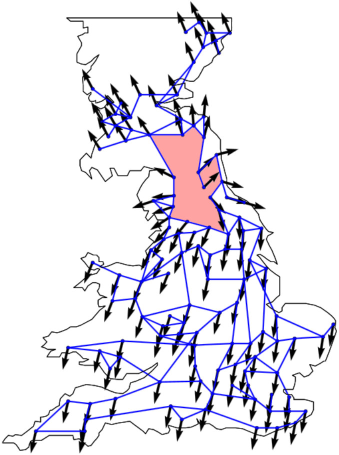
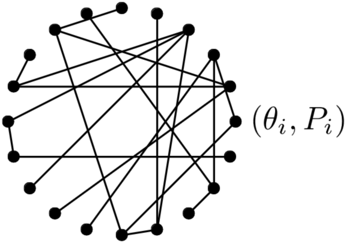
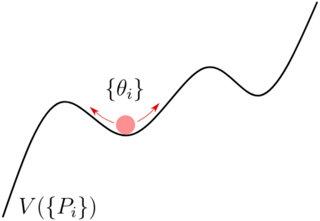
# Synchronization on networks



Delabays, MT, Jacquod, Chaos **27**(10), 103109 (2017)

# Robustness of synchronous networks



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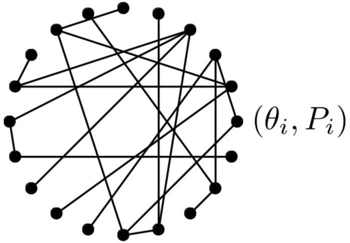
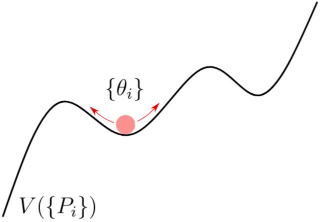
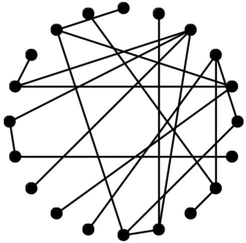
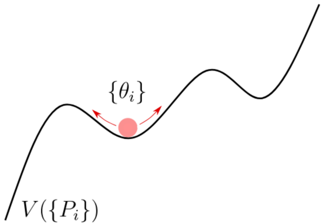



- Size of the basin of attraction

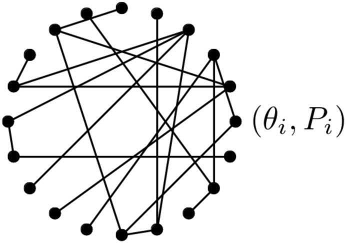
# Robustness of synchronous networks

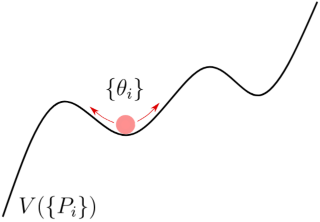



- Size of the basin of attraction
- Near equilibrium dynamics

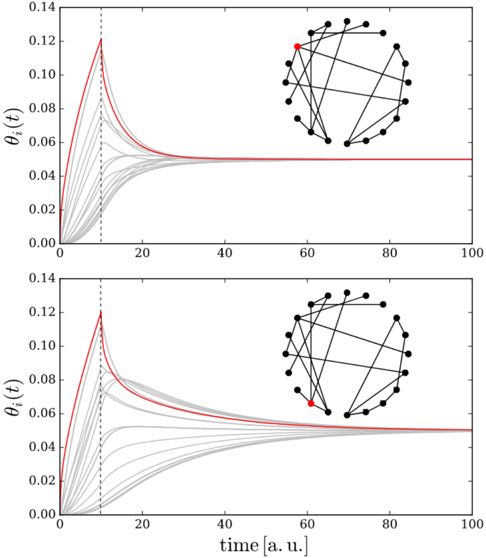
# Robustness of synchronous networks




- Size of the basin of attraction
- Near equilibrium dynamics
- Transitions between fixed points

# Robustness of synchronous networks



# Robustness of synchronous networks to noise inputs

**Near equilibrium dynamics**

$$0 = \omega_i - \sum_{j=1}^{N} b_{ij} \sin(\theta_i^{(0)} - \theta_j^{(0)})\,, \text{for } i = 1, ..., N\,. \tag{5}$$

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$$\delta\dot{\theta}_i = -\sum_{j=1}^{N} b_{ij} \cos(\theta_i^{(0)} - \theta_j^{(0)})(\delta\theta_i - \delta\theta_j) + \eta_i(t)\,, \text{for } i = 1, ..., N\,. \tag{6}$$

$\eta_i(t)$: Noise inputs.

# Robustness of synchronous networks to noise inputs

**Near equilibrium dynamics**

$$\boldsymbol{\delta\dot{\theta}} = -\mathbb{L}(\{\theta_k^{(0)}\})\boldsymbol{\delta\theta} + \boldsymbol{\eta}(t)\,, \text{for } i = 1, ..., N\,. \tag{7}$$

$\mathbb{L}(\{\theta_k^{(0)}\})$: Jacobian, with eigenvalues $\lambda_1 = 0 < \lambda_2 \leq ... \leq \lambda_N$.

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**Solution**

$$\delta\theta_i(t) = \sum_\alpha e^{-\lambda_\alpha\, t} \int_0^t e^{\lambda_\alpha\, t'} \sum_j \eta_j(t') u_{\alpha,j} \mathrm{d}t'\, u_{\alpha,i}\,. \tag{8}$$

# Robustness of synchronous networks to noise inputs

**Uncorrelated white noise**

$$\langle \eta_i(t)\eta_j(t')\rangle = \eta_0^2\,\tau_0\,\delta_{ij}\,\delta(t-t') \tag{9}$$

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**Variance at node** $i$

$$\langle \delta\theta_i^2\rangle = \frac{\eta_0^2\,\tau_0}{2}\sum_\alpha \frac{u_{\alpha,i}^2}{\lambda_\alpha} \tag{10}$$

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**Average variance in the network**

$$N^{-1}\sum_i \langle \delta\theta_i^2\rangle = \frac{\eta_0^2\,\tau_0}{2N}\sum_\alpha \frac{1}{\lambda_\alpha} \tag{11}$$

# Robustness of synchronous networks to noise inputs

**Time-correlated noise**

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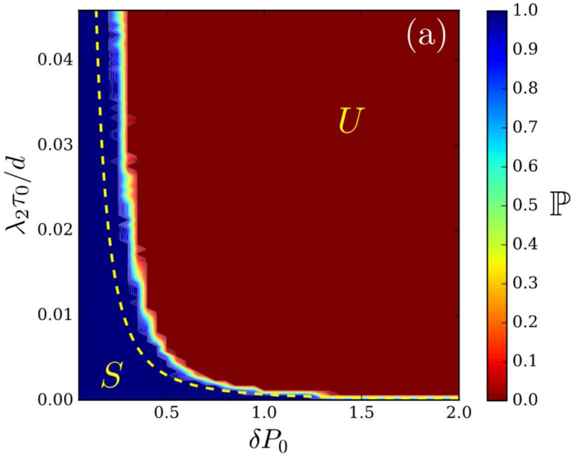
**Average variance in the network**

$$N^{-1}\sum_i \langle \delta\theta_i^2\rangle = \frac{\eta_0^2}{N}\sum_\alpha \frac{1}{\lambda_\alpha^2} \tag{14}$$

**Escape from the initial basin of attraction**




MT, Delabays, Jacquod, Phys. Rev. E **99** (6), 062213 (2019)

# Robustness of synchronous networks to noise inputs

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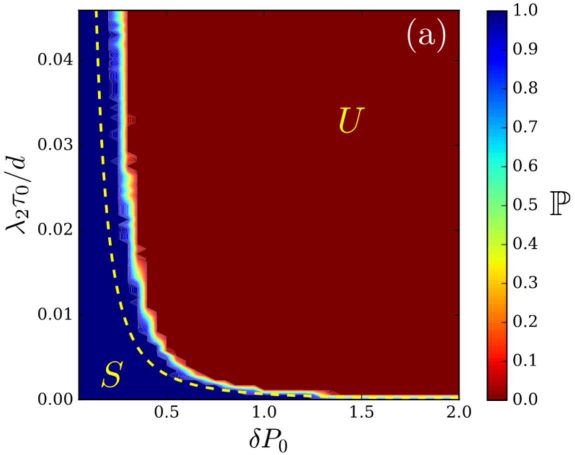


**It can be even worst!**

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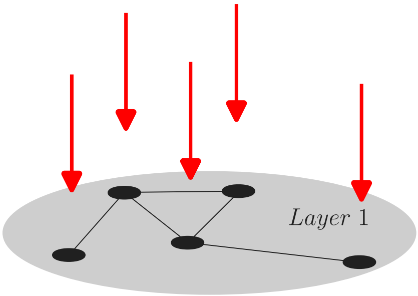
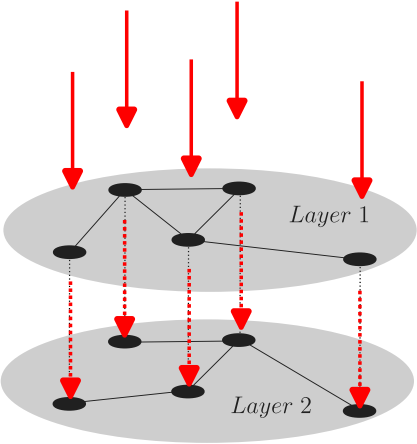
MT, Delabays, Jacquod, Phys. Rev. E **99** (6), 062213 (2019)

# Layered Networks




MT, J. Phys. Complex. **3**, 03LT01 (2022)
MT, Chaos **32**(12), 121102, *fast track* (2022)

# Layered Networks

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# Layered Networks: Multistability

**Layered Kuramoto oscillators**:

$$\dot{\phi}_i = \omega_i^{(1)} - \sum_{j=1}^{N} b_{ij}^{(1)} \sin(\phi_i - \phi_j) + \eta_i \quad i = 1, ...N \,,$$
$$\dot{\theta}_i = \omega_i^{(2)} - \sum_{j=1}^{N} b_{ij}^{(2)} \sin(\theta_i - \theta_j) + f_i(\{\phi_k\}, \{\theta_k\}) \quad i = 1, ...N, , \qquad (15)$$

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$$\mathbb{L}_{ij}^{(1)}(\{\phi_i^{(0)}\}) = \begin{cases} -b_{ij}^{(1)} \cos(\phi_i^{(0)} - \phi_j^{(0)}) \,, & i \neq j \,, \\ \sum_k b_{ik}^{(1)} \cos(\phi_i^{(0)} - \phi_k^{(0)}) \,, & i = j \,, \end{cases}$$

$$\mathbb{L}_{ij}^{(2)}(\{\theta_i^{(0)}\}) = \begin{cases} -b_{ij}^{(2)} \cos(\theta_i^{(0)} - \theta_j^{(0)}) \,, & i \neq j \,, \\ \sum_k b_{ik}^{(2)} \cos(\theta_i^{(0)} - \theta_k^{(0)}) \,, & i = j \,, \end{cases}$$

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Two sets of time-scales:

$$\lambda_1^{(1)} = 0 < \lambda_2^{(1)} \leq \ldots \leq \lambda_N^{(1)} \tag{16}$$

$$\lambda_1^{(2)} = 0 < \lambda_2^{(2)} \leq \ldots \leq \lambda_N^{(2)} \tag{17}$$

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Uncorrelated white noise: $\langle \eta_i(t)\eta_j(t')\rangle = \eta_0^2\, \delta_{ij}\, \delta(t - t')$

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Simplest choice: $f_i(\{\phi_k\}, \{\theta_k\}) = d(\phi_i - N^{-1} \sum_j \phi_j)$

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Noise in the 2nd layer: $\langle \phi_i(t) \phi_j(t') \rangle = \frac{\eta_0^2}{2} \sum_\alpha \frac{u_{\alpha,i}^{(1)} u_{\alpha,j}^{(1)}}{\lambda_\alpha^{(1)}} e^{-\lambda_\alpha^{(1)} |t - t'|}$.

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MT, J. Phys. Complex. **3**, 03LT01 (2022)
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# Layered Networks

**Analytical treatment**:

$$\phi_i(t) = \sum_\alpha e^{-\lambda_\alpha^{(1)} t} \int_0^t e^{\lambda_\alpha^{(1)} t'} \sum_j \eta_j \, u_{\alpha,j}^{(1)} \, \mathrm{d}t' \, u_{\alpha,i}^{(1)} \,, \tag{16}$$

$$\theta_i(t) = \sum_\alpha e^{-\lambda_\alpha^{(2)} t} \int_0^t e^{\lambda_\alpha^{(2)} t'} \sum_j \phi_j \, u_{\alpha,j}^{(2)} \, \mathrm{d}t' \, u_{\alpha,i}^{(2)} \,. \tag{17}$$

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**Layer 1**:

$$\langle \phi_i^2 \rangle = \frac{\eta_0^2}{2} \sum_\alpha \frac{{u_{\alpha,i}^{(1)}}^2}{\lambda_\alpha^{(1)}} \,, \qquad (18)$$

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**Layer 2**:

$$\langle \theta_i^2 \rangle = \frac{\eta_0^2}{2} \sum_{\alpha,\beta,\gamma} \sum_{k,l} \frac{u_{\gamma,k}^{(1)} u_{\gamma,l}^{(1)} u_{\alpha,k}^{(2)} u_{\beta,l}^{(2)} [2\lambda_\gamma^{(1)} + \lambda_\alpha^{(2)} + \lambda_\beta^{(2)}]}{\lambda_\gamma^{(1)} (\lambda_\alpha^{(2)} + \lambda_\beta^{(2)}) (\lambda_\gamma^{(1)} + \lambda_\alpha^{(2)}) (\lambda_\gamma^{(1)} + \lambda_\beta^{(2)})} u_{\alpha,i}^{(2)} u_{\beta,i}^{(2)} \,. \tag{19}$$

**Analytical treatment**:

$$\phi_i(t) = \sum_\alpha e^{-\lambda_\alpha^{(1)} t} \int_0^t e^{\lambda_\alpha^{(1)} t'} \sum_j \eta_j \, u_{\alpha,j}^{(1)} \, \mathrm{d}t' \, u_{\alpha,i}^{(1)} \,, \tag{16}$$

$$\theta_i(t) = \sum_\alpha e^{-\lambda_\alpha^{(2)} t} \int_0^t e^{\lambda_\alpha^{(2)} t'} \sum_j \phi_j \, u_{\alpha,j}^{(2)} \, \mathrm{d}t' \, u_{\alpha,i}^{(2)} \,. \tag{17}$$

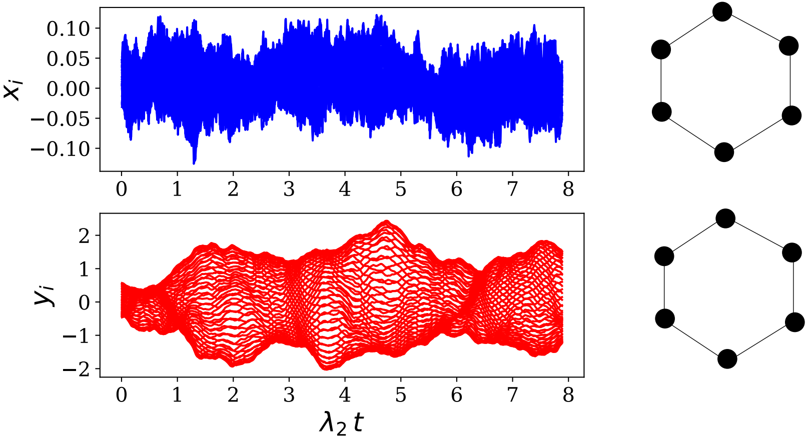
**Layer 1**:

$$\langle \phi_i^2 \rangle = \frac{\eta_0^2}{2} \sum_\alpha \frac{{u_{\alpha,i}^{(1)}}^2}{\lambda_\alpha^{(1)}} \,, \tag{18}$$

**Layer 2**: Same networks

$$\langle \theta_i^2 \rangle = \frac{\eta_0^2}{4} \sum_\alpha \frac{u_{\alpha,i}^2}{\lambda_\alpha^3} \,. \tag{19}$$

# Layered Networks: Amplification

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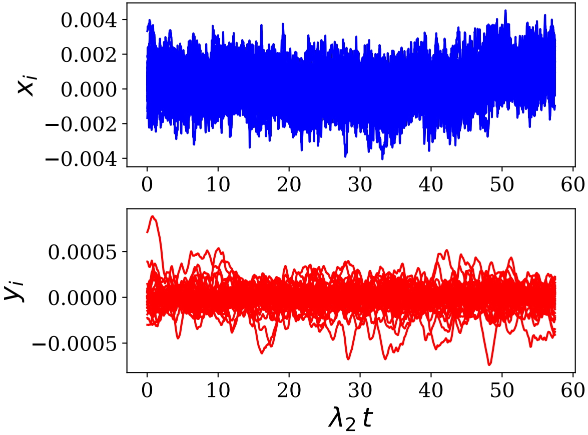
# Layered Networks: Amplification

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# Layered Networks: Transitions

$$\langle x_i^2 \rangle = \frac{\eta_0^2}{2} \sum_\alpha \frac{{u_{\alpha,i}^{(1)}}^2}{\lambda_\alpha^{(1)}}\,, \langle y_i^2 \rangle = \frac{\eta_0^2}{4} \sum_\alpha \frac{u_{\alpha,i}^2}{\lambda_\alpha^3}\,.$$

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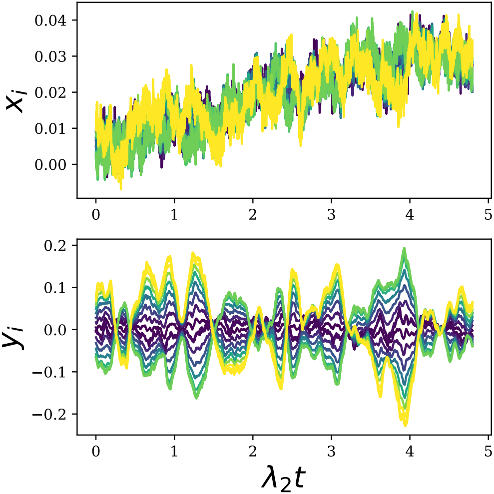


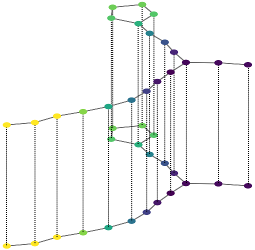

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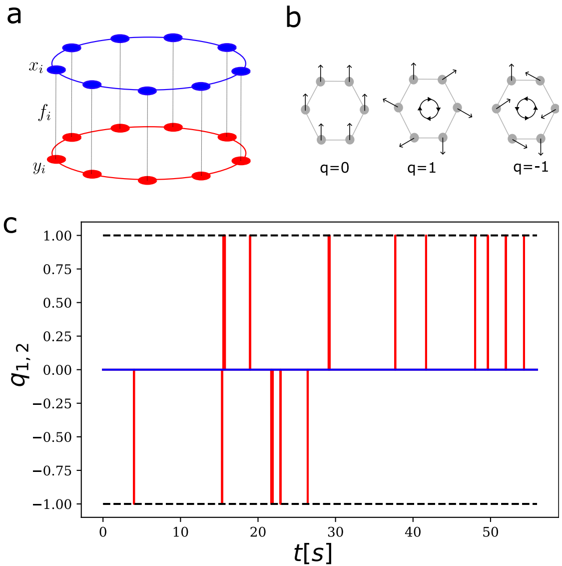
# Layered Networks: Transitions

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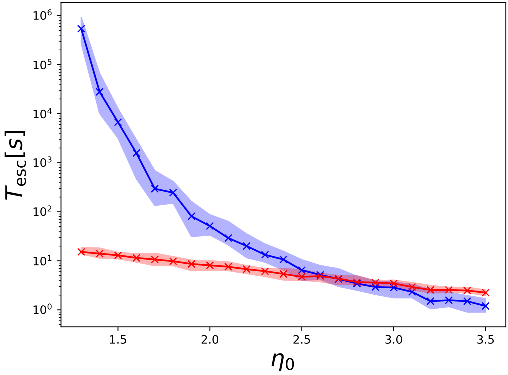
# Layered Networks: Transitions

**Cycle**

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# Layered Networks: Transitions

Rescaled noise: $\boldsymbol{\eta} = d\,\overline{\boldsymbol{\phi}}$

$$N^{-1}\sum_i \langle \xi_i^2 \rangle = \eta_0^2\,, \tag{20}$$

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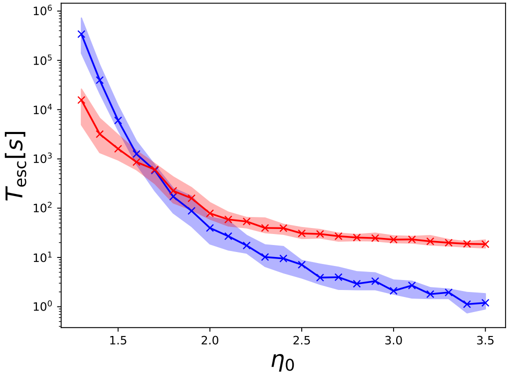


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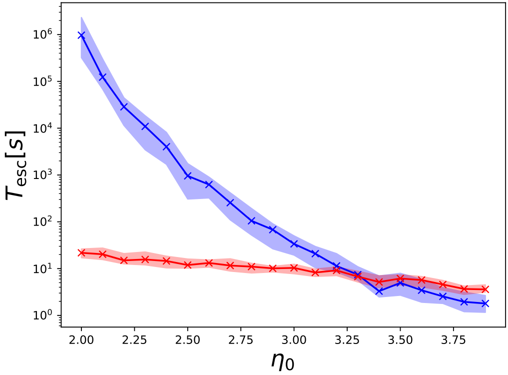
# Layered Networks: Transitions

**Watts-Strogatz**

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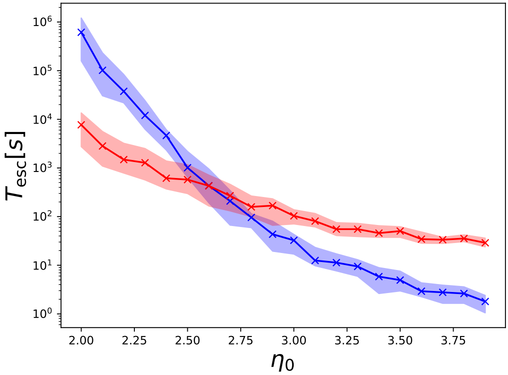
# Layered Networks: Transitions

Rescaled noise: $\eta = d\,\overline{\delta \mathbf{x}}$

$$N^{-1} \sum_i \langle \xi_i^2 \rangle = \eta_0^2\,, \tag{21}$$

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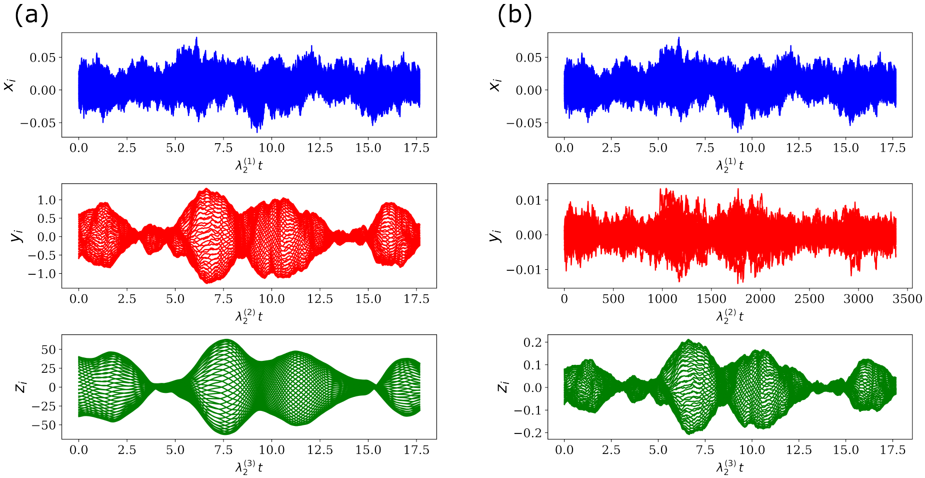
# Layered Networks: Amplification

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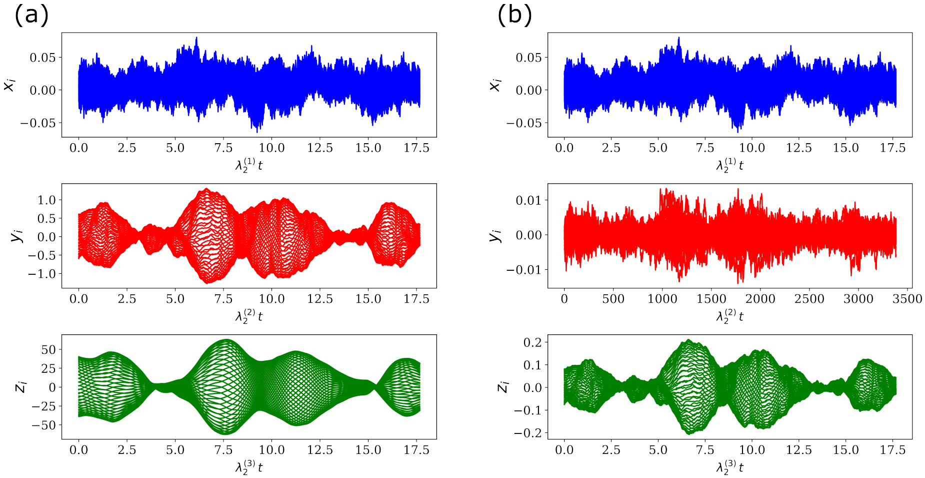
# Layered Networks: Amplification




Applications to photonics and brain dynamics...?

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# Conclusion and future work

**So far**

- White-noise is not that dangerous... compared to correlated noise.
- Be careful with system-specific correlations.

# Conclusion and future work

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- White-noise is not that dangerous... compared to correlated noise.
- Be careful with system-specific correlations.

**Future work**

- Spatial correlation (Collaboration with NRL).