Including the magnitude variability in the ordinal pattern analysis

Joint work with Nicolás Rubido, University of Aberdeen.

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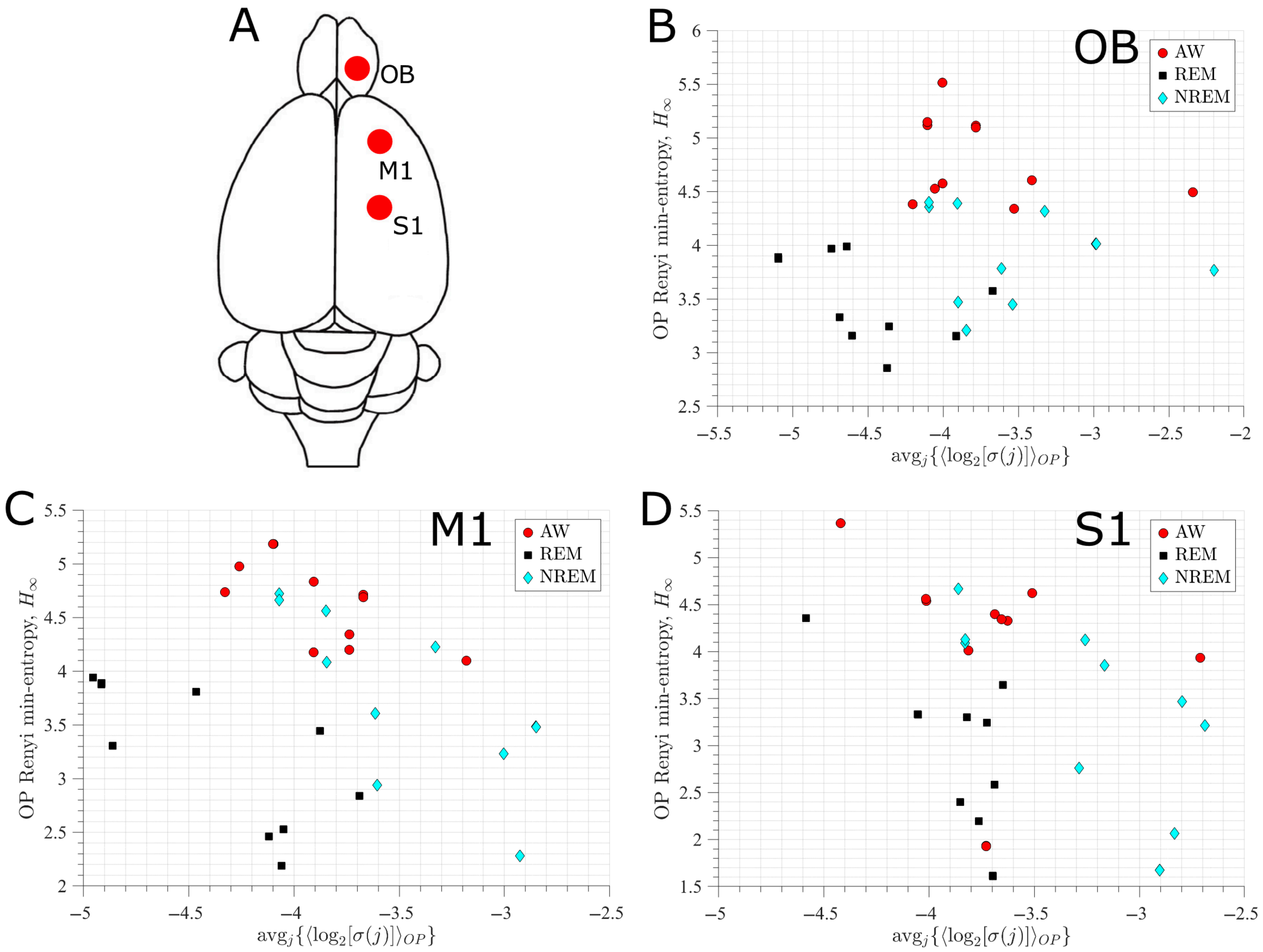
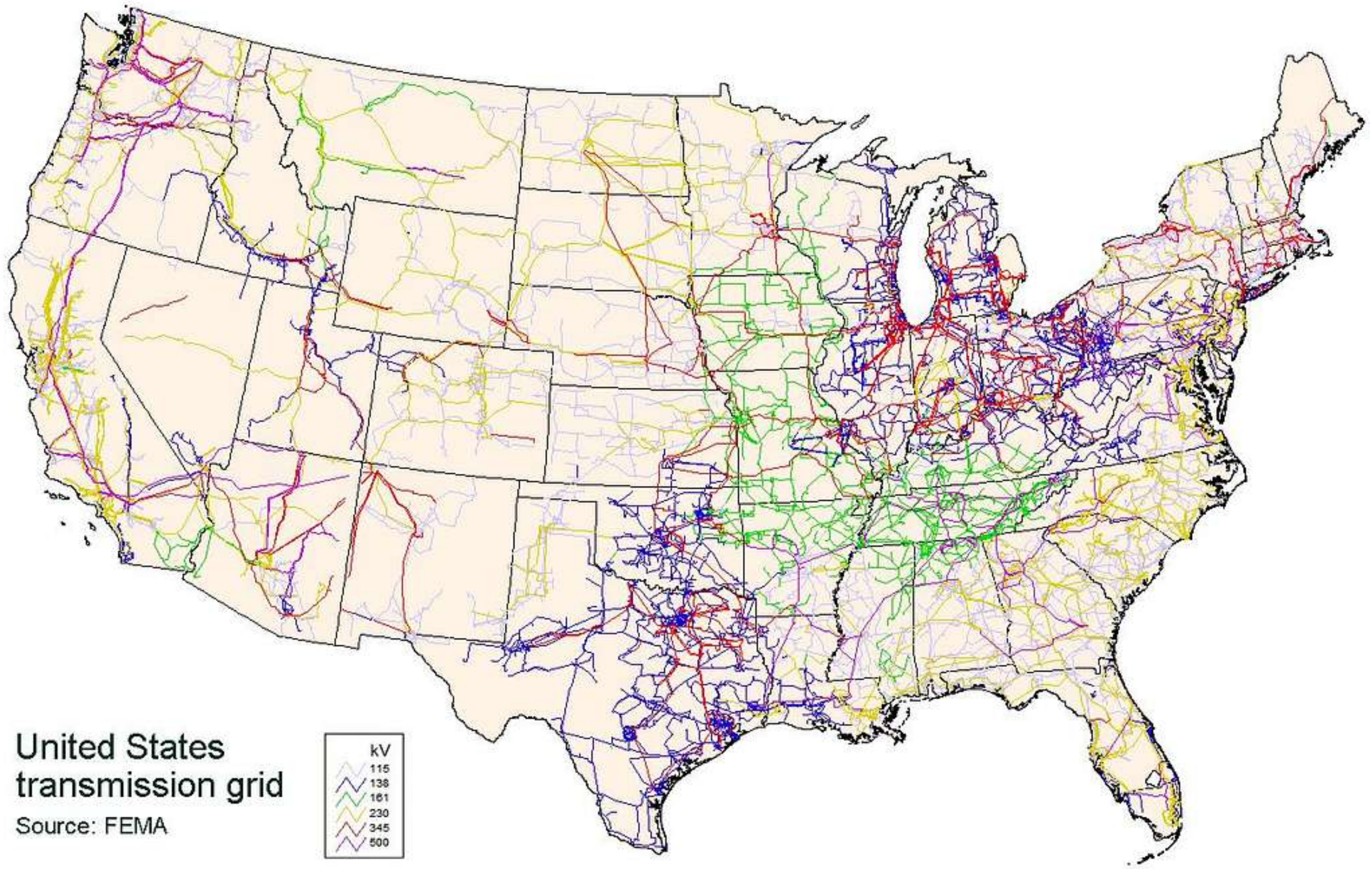
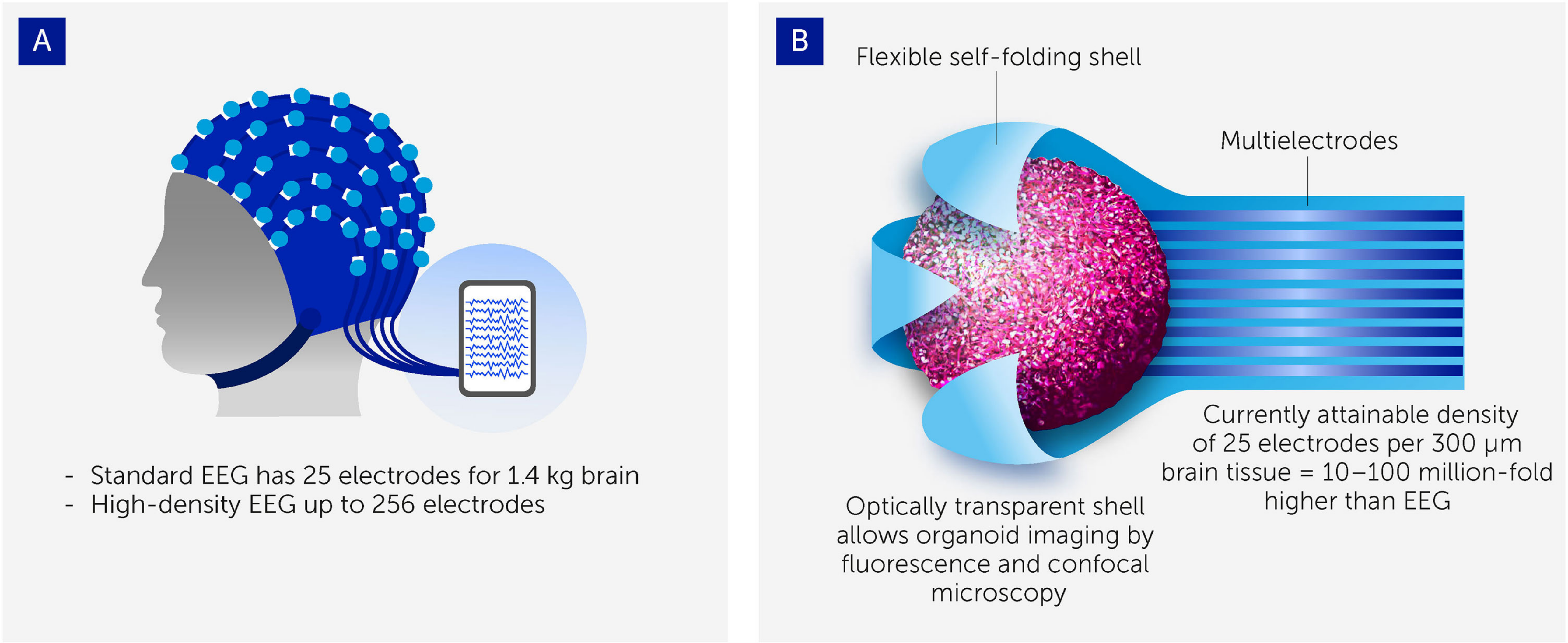
MT, González, Rubido, *Entropy* **27**, 840 (2025)



Motivation

More and more data…

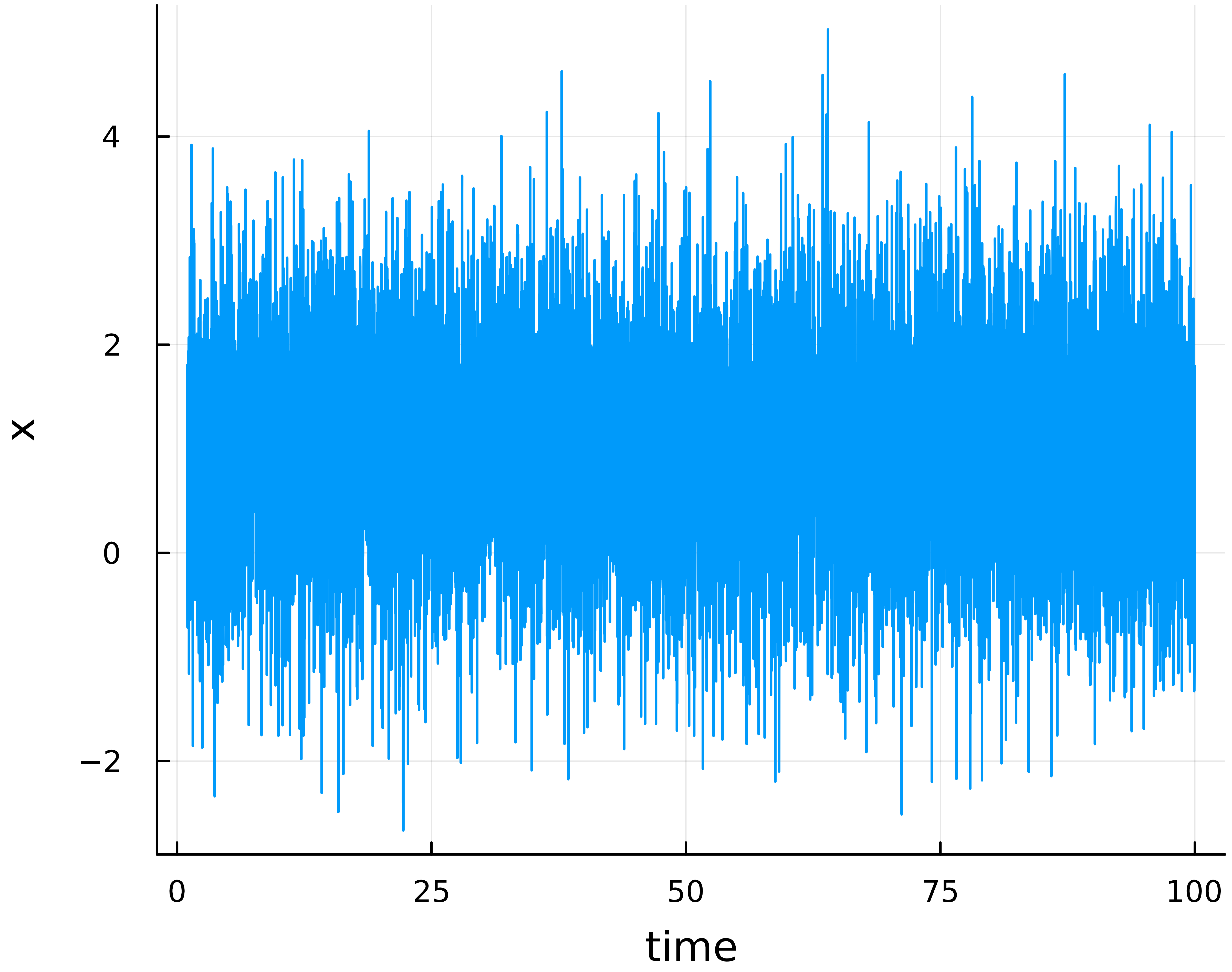




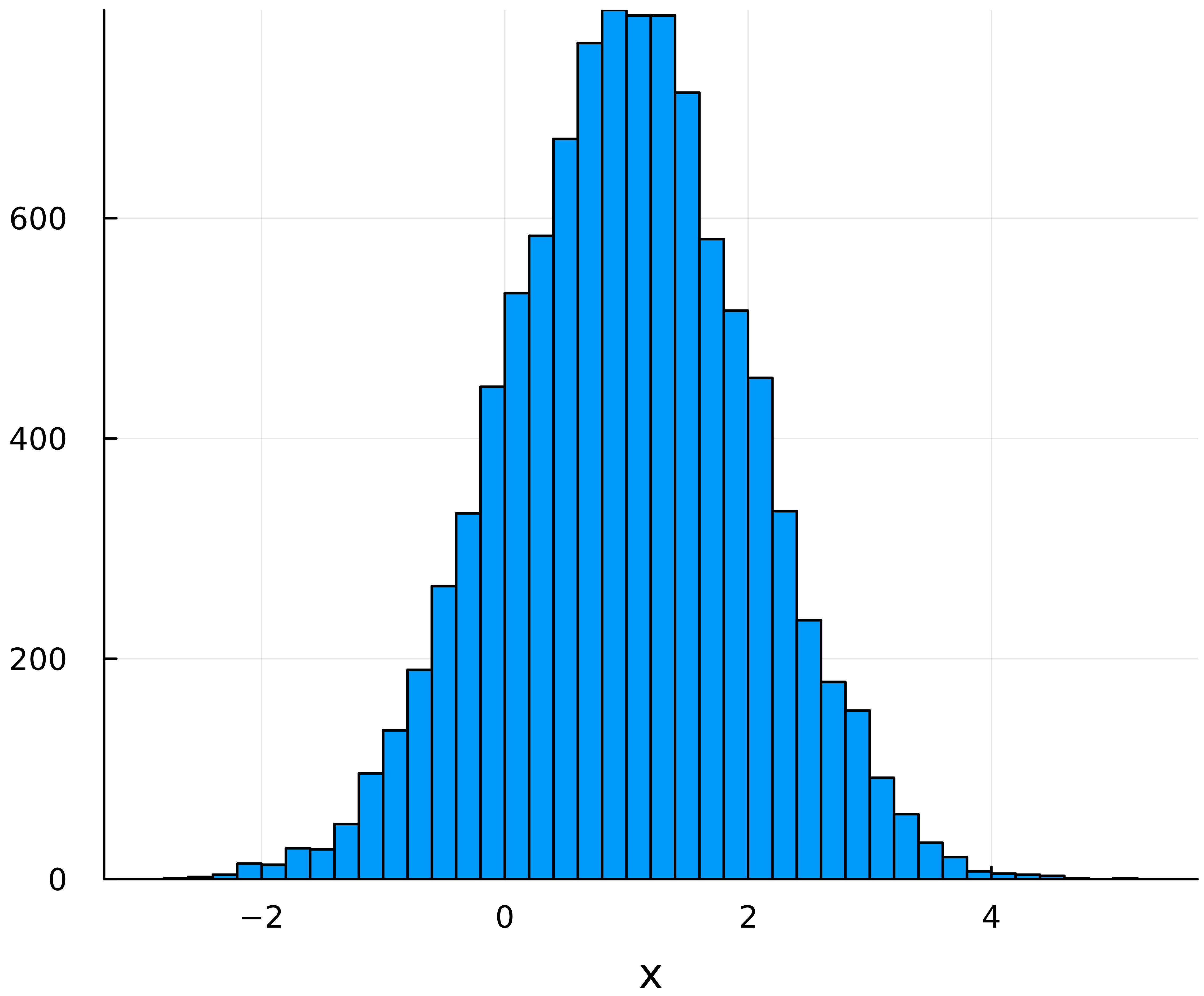
Front Sci 1:1017235 (2023).

Motivation

Time-series

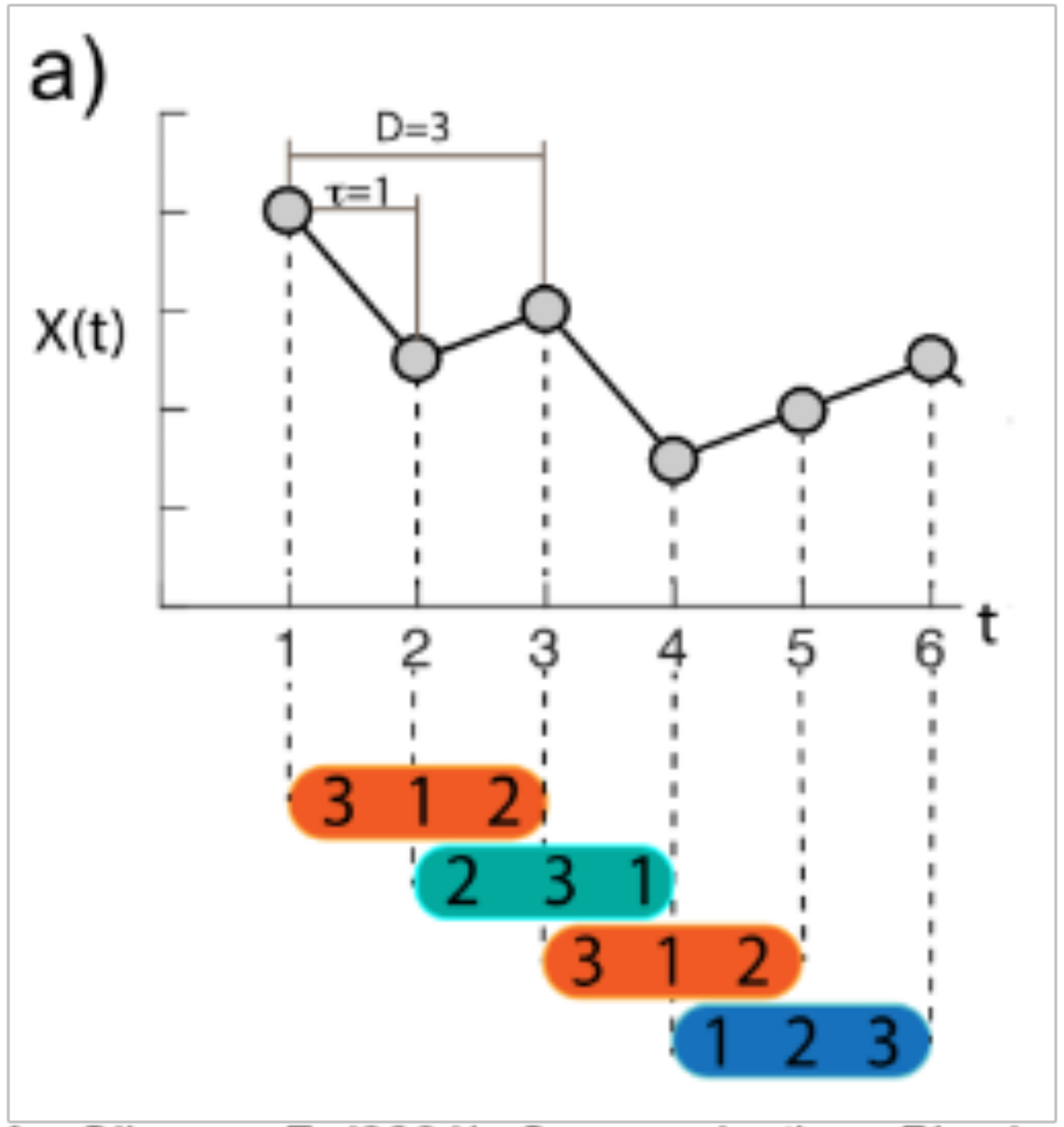


Throws away the time-ordering



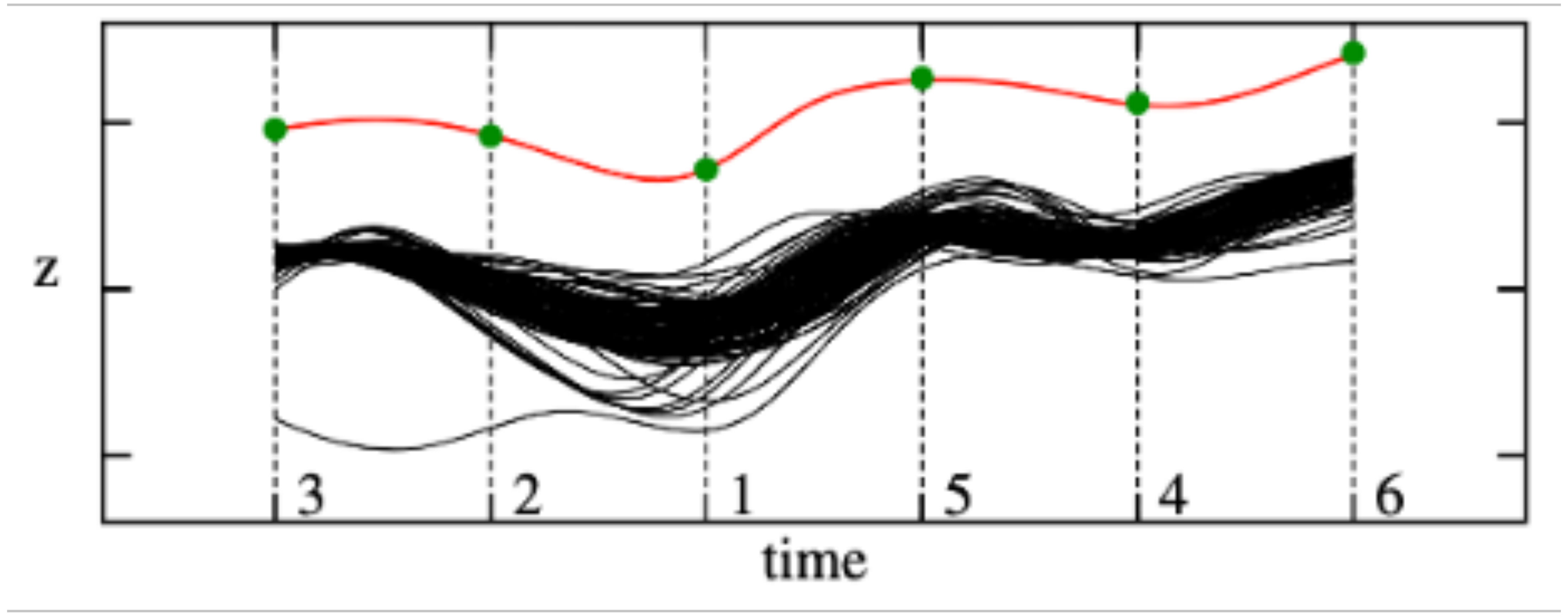
Motivation

Finding patterns in time-series



Zanin, M., Olivares, F. (2021). *Communications Physics*, *4*(1), 190.

Some information is thrown away!



Politi, A. (2017. *Physical review letters*, *118*(14), 144101.

Include the magnitude variability back

Definitions

Finding patterns in time-series (Bandt-Pompe, 2002)

$$\{x_t\}_{t=1}^{T} = \{x_1, x_2, \ldots, x_T\}$$

x_t is the magnitude at the discrete time index $t \in \mathbb{N}$,

x_1 is the initial state, and T is the length of the signal.

$$\{x_1, x_2, \ldots, x_T\} \mapsto \{x_1, x_{1+\tau}, \ldots, x_{1+n\tau}\}$$

$n = \lfloor (T-1)/\tau \rfloor$ is the smallest integer closest to $(T-1)/\tau$.

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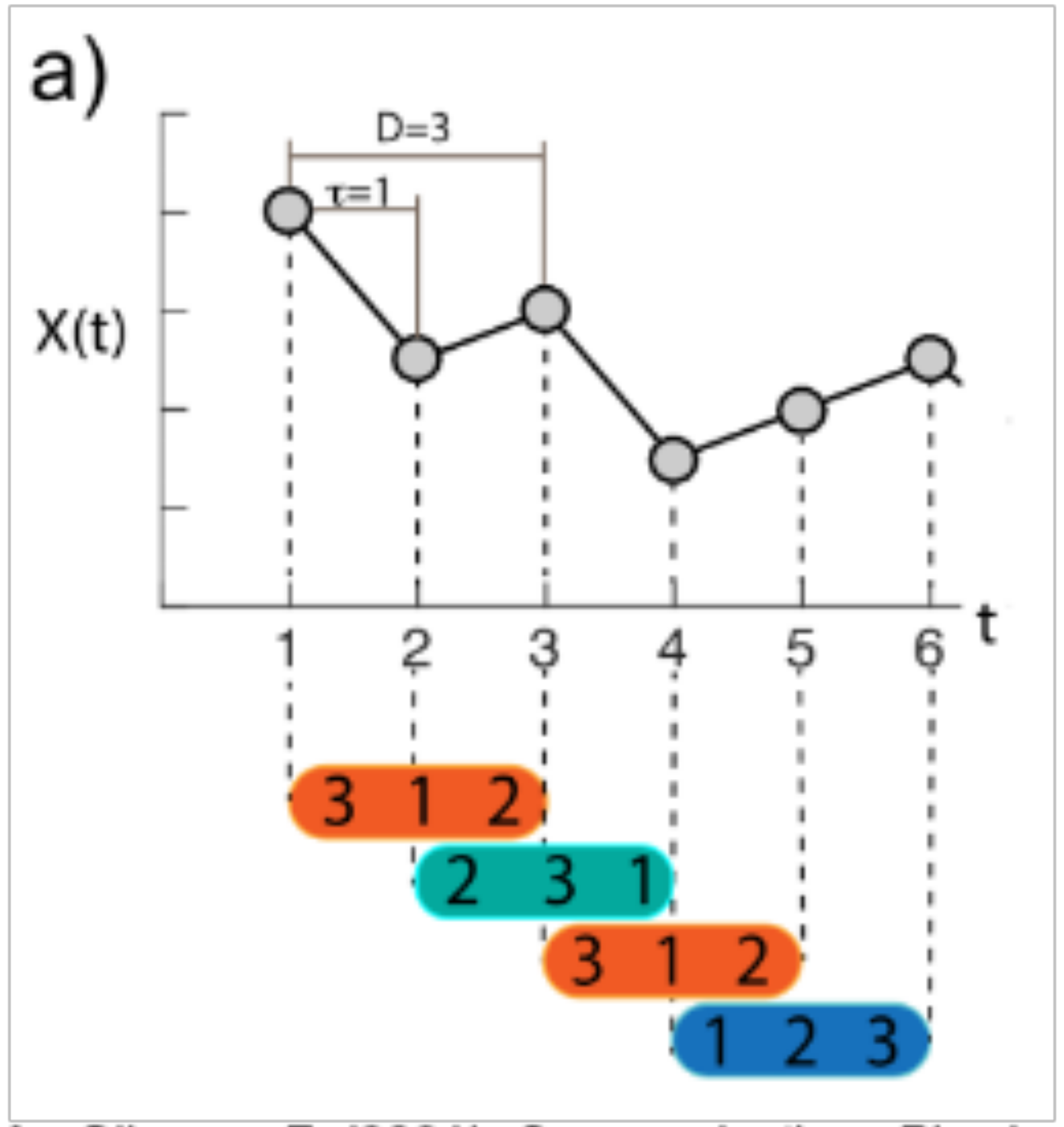

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$\{x_t\}_{t=1}^{T}$ becomes $\{x_1, \ldots, x_D\}, \{x_D, \ldots, x_{2D-1}\}, \{x_{2D-1}, \ldots, x_{3D-2}\}, \ldots, \{x_{(m-1)(D-1)+1}, \ldots, x_{m(D-1)}\}$,
where $m = \lceil T/(D-1) \rceil$ is the largest integer closest to $T/(D-1)$.

Definitions

Information theory (Shannon, 1948)

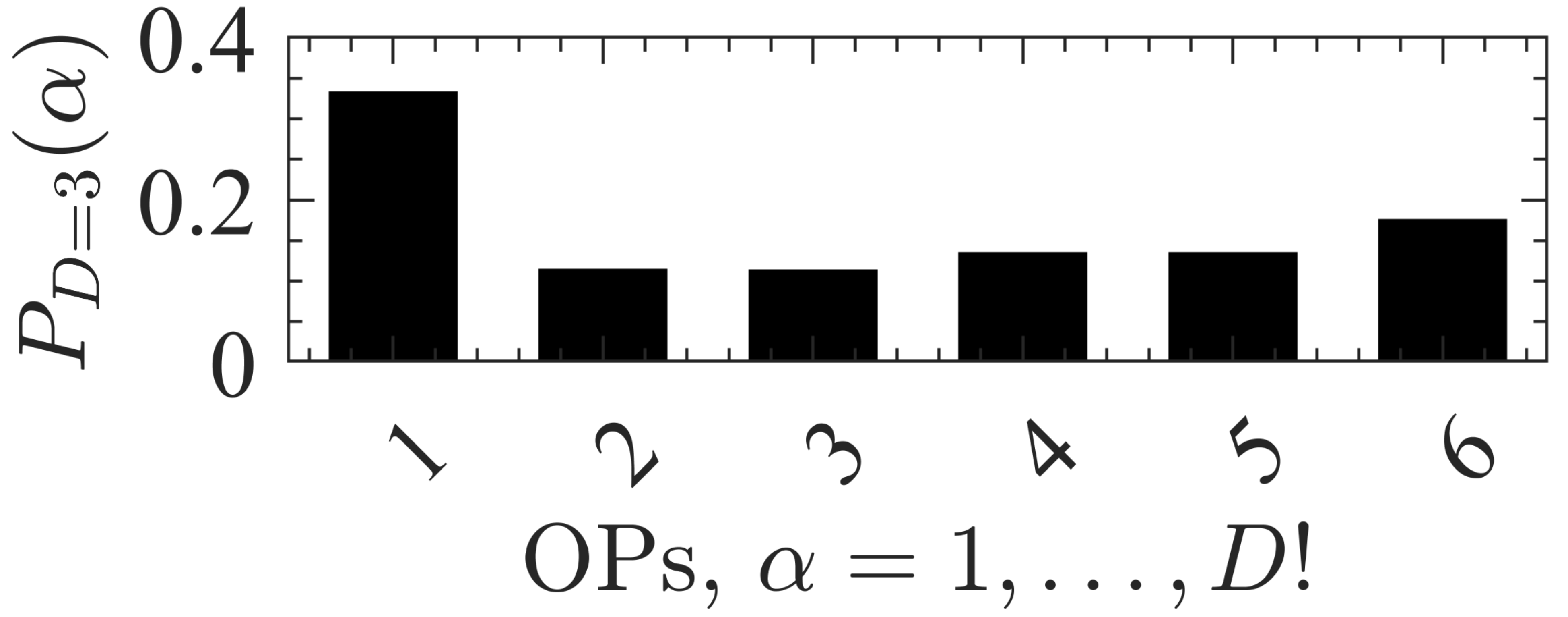
Information content of an ordinal patern $\log\left[1/P(\alpha)\right]$

$P(\alpha)$ is the probability of having OP α .

Permutation entropy (Bandt-Pompe, 2002)

$$H = \sum_{\alpha=1}^{D!} P(\alpha)\,\log_2\left[\frac{1}{P(\alpha)}\right] = \left\langle \log_2\left[\frac{1}{P(\alpha)}\right]\right\rangle .$$

The maximum value of H is $H_{\max} = \log_2(D!)$.



Rényi min-entropy

$$H_\infty = \lim_{q\to\infty} \frac{1}{1-q} \log_2\left[\sum_{\alpha=1}^{D!} P^q(\alpha)\right] = \min_\alpha \left\{\log_2\left[\frac{1}{P(\alpha)}\right]\right\} = -\log_2\left[\max_\alpha\{P(\alpha)\}\right] .$$

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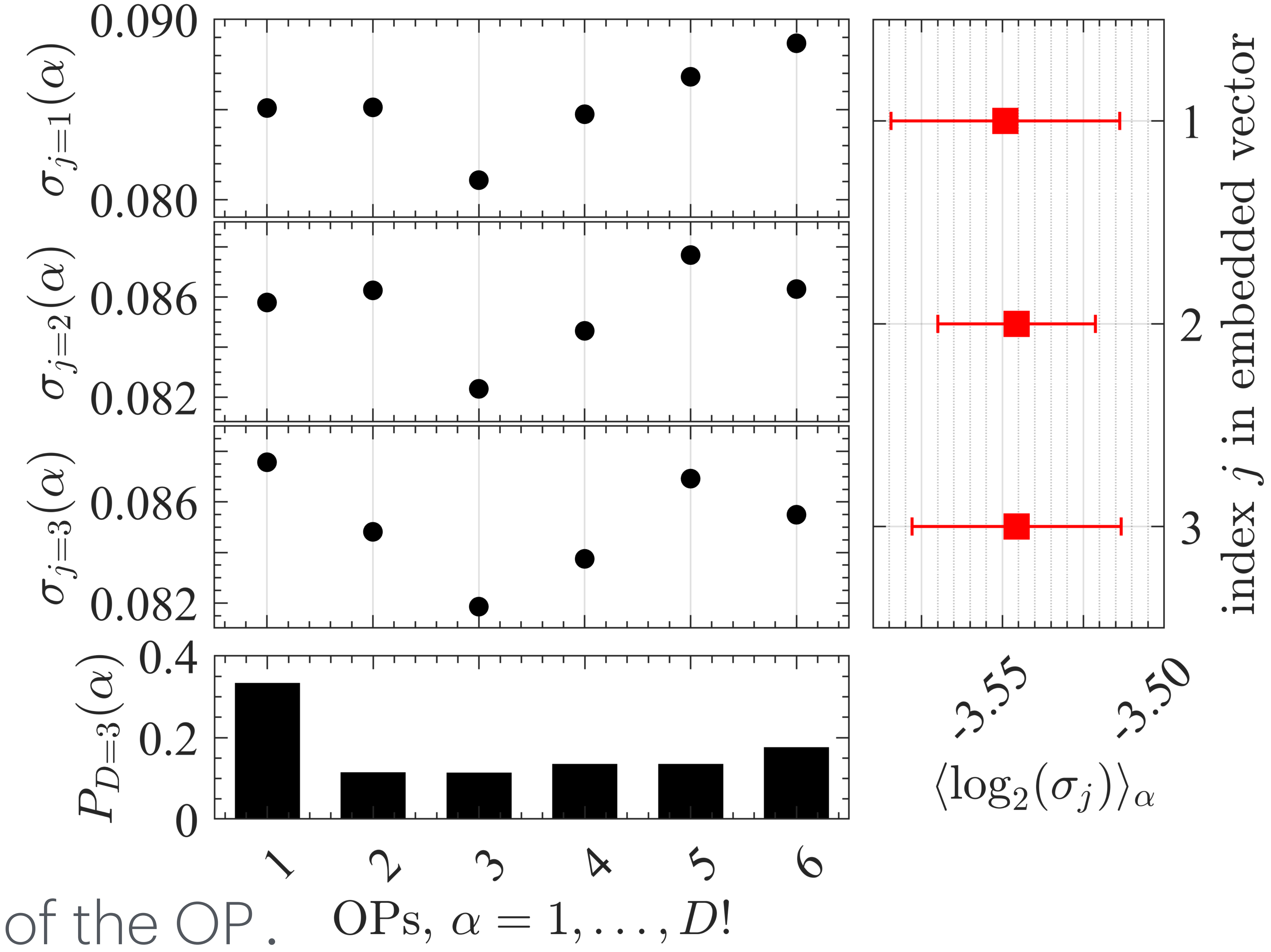
Signal variability (Politi, 2017)

$$\left\langle \log_2\left[\sigma_j\right] \right\rangle = \sum_{\alpha=1}^{D!} P(\alpha) \log_2\left[\sigma_j(\alpha)\right]$$

with $\sigma_j(\alpha)$ the standard deviation at the jth position of the OP.



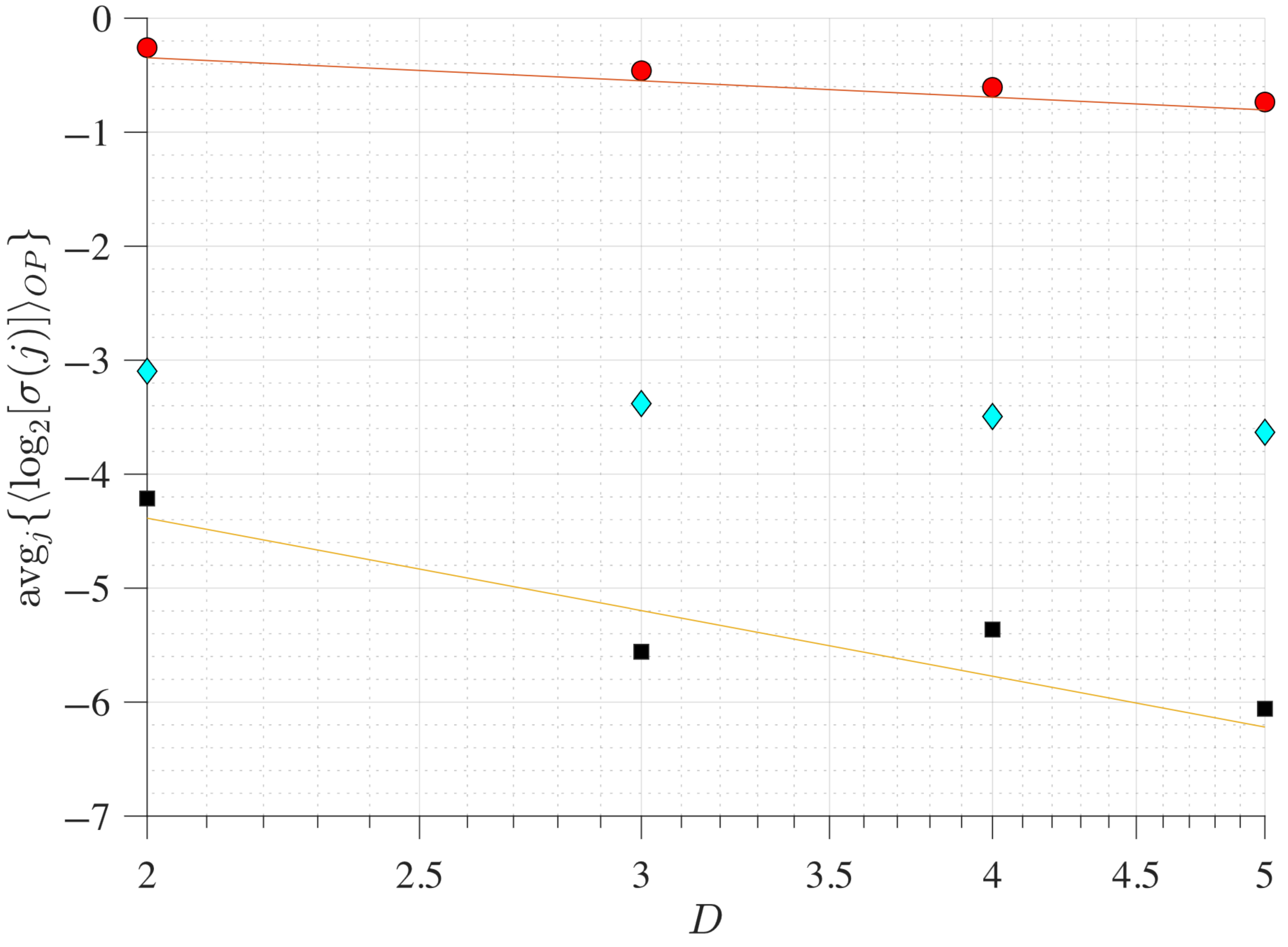
Signal variability

Signal variability (Politi, 2017)

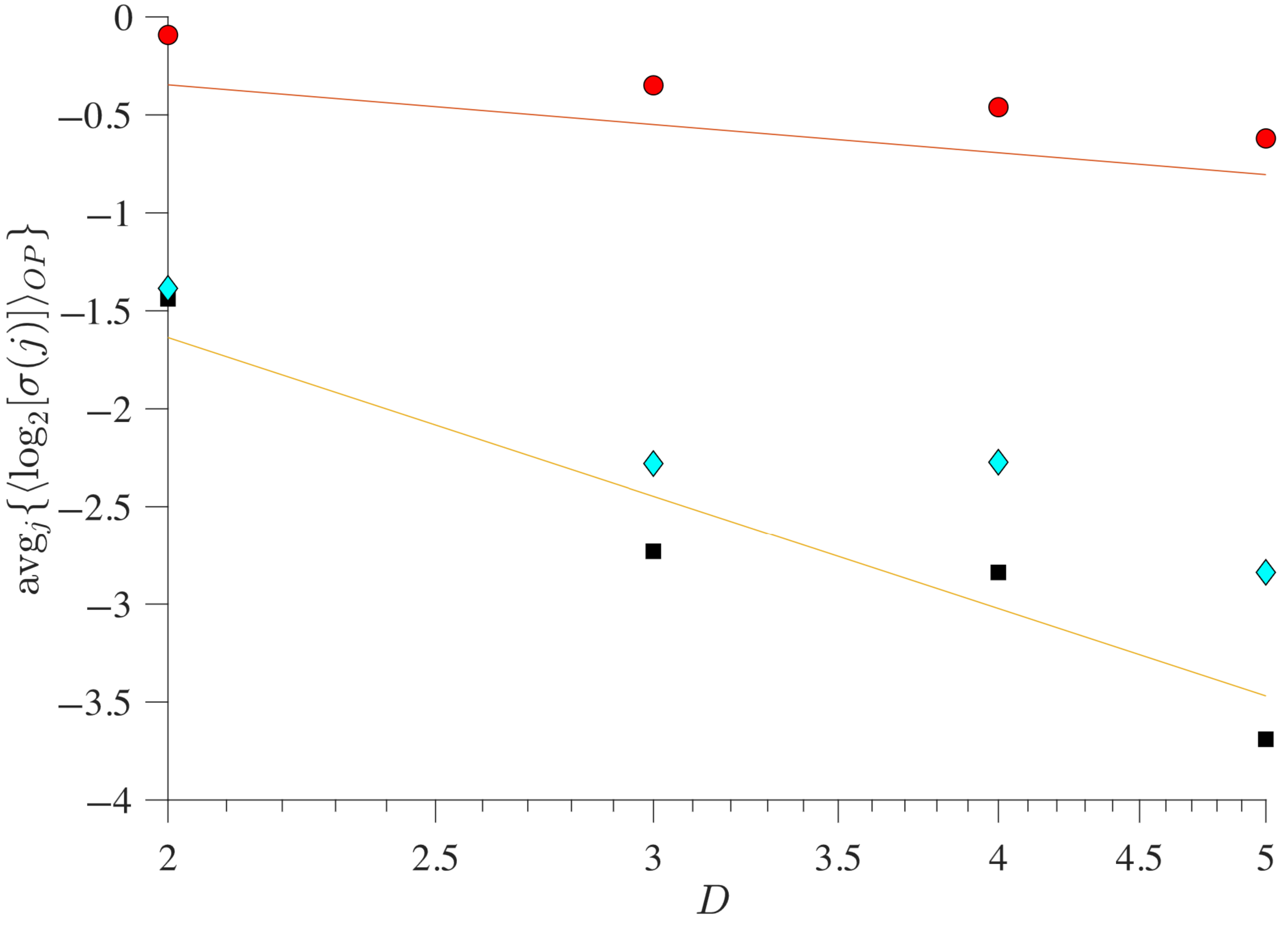
$$\left\langle \log_2 \left[\sigma_j\right] \right\rangle = \sum_{\alpha=1}^{D!} P(\alpha) \log_2 \left[\sigma_j(\alpha)\right]$$

Identify noisy signal

Bivariate logistic map



Hénon map



Signal variability

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$$\left\langle \log_2 \left[\sigma_j\right]\right\rangle = \sum_{\alpha=1}^{D!} P(\alpha)\log_2\left[\sigma_j(\alpha)\right]$$

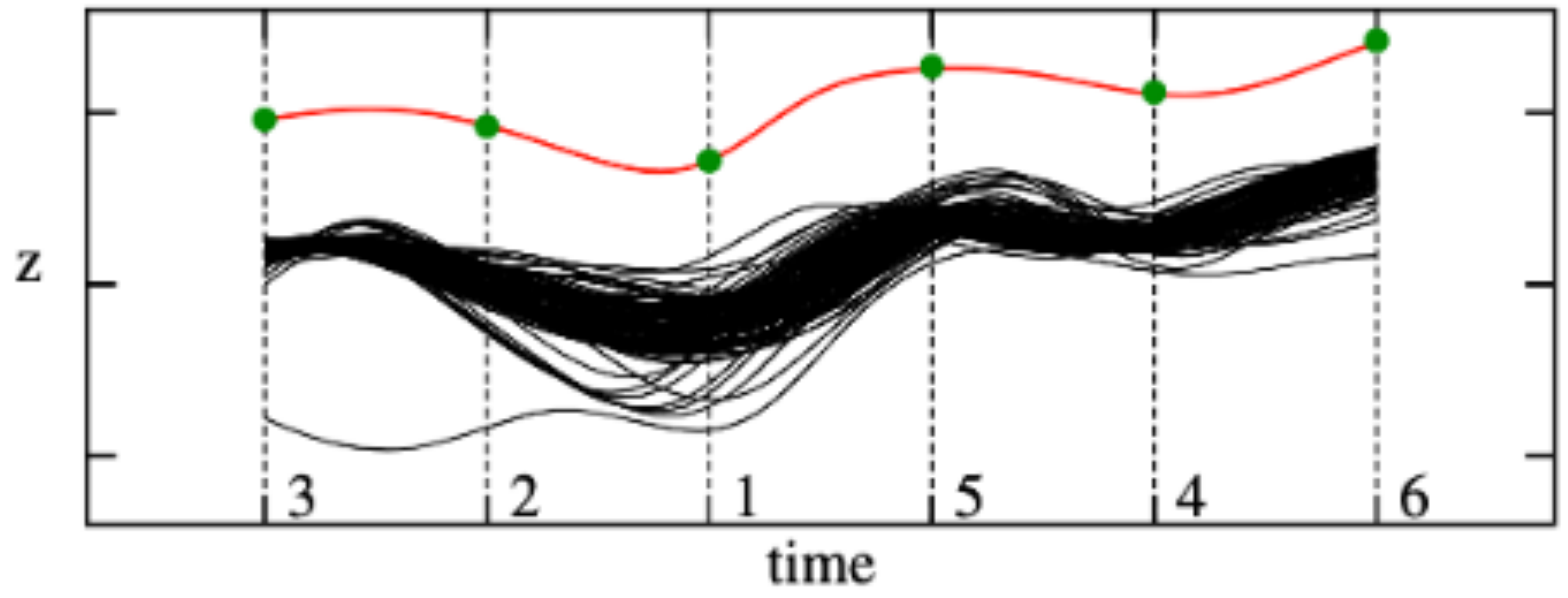
Kolmogorov-Sinai entropy (Politi, 2017)

Relative entropy $\tilde{H}(D) = H(D) + d\langle \log_2 \sigma_j(D)\rangle$

$$h_{KS} = [\tilde{H}(D+1) - \tilde{H}(D)]/T$$

Information dimension *d…*

Include the signal variability as an additional descriptor.



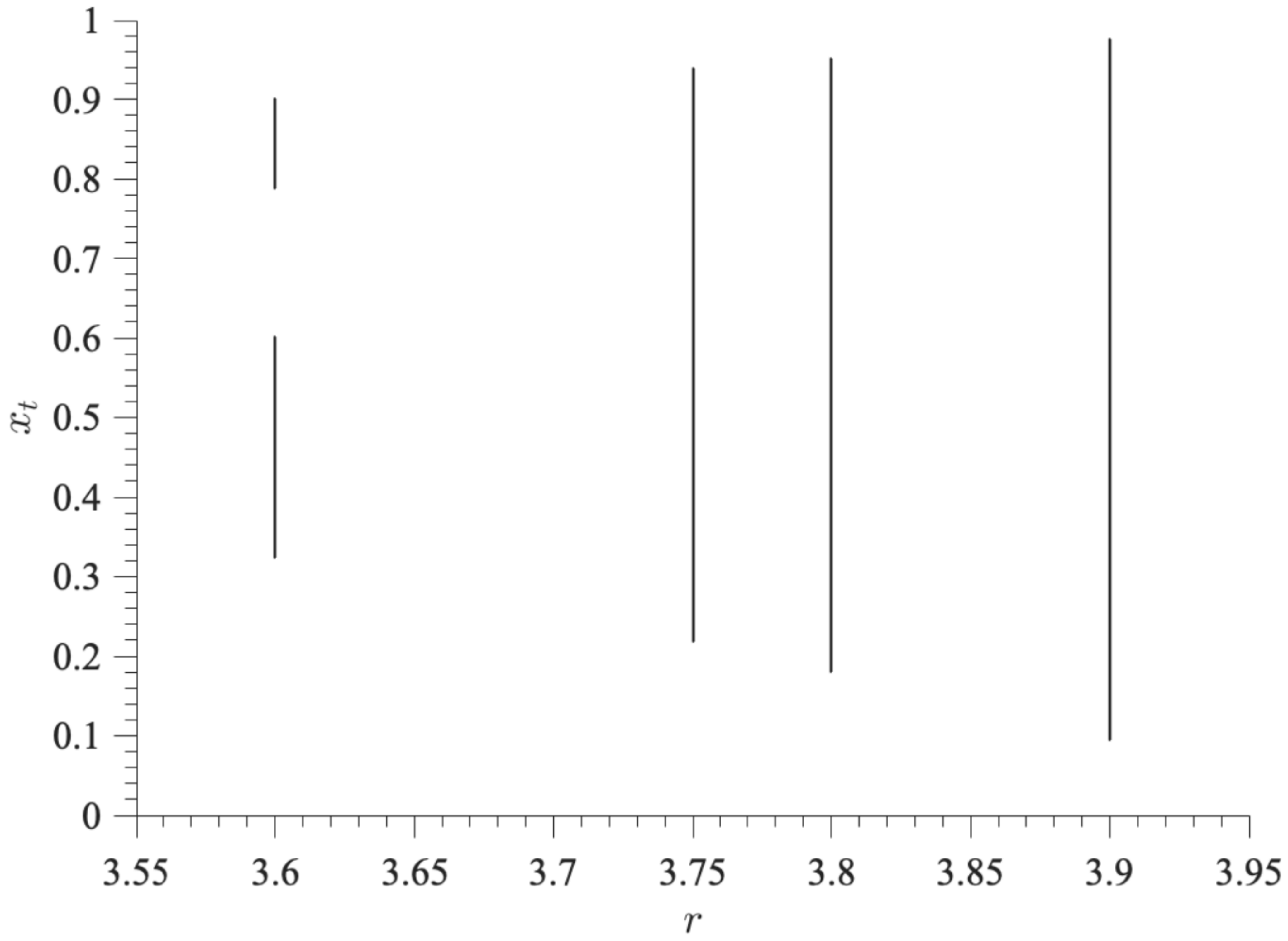
Politi, A. (2017. *Physical review letters*, *118*(14), 144101.

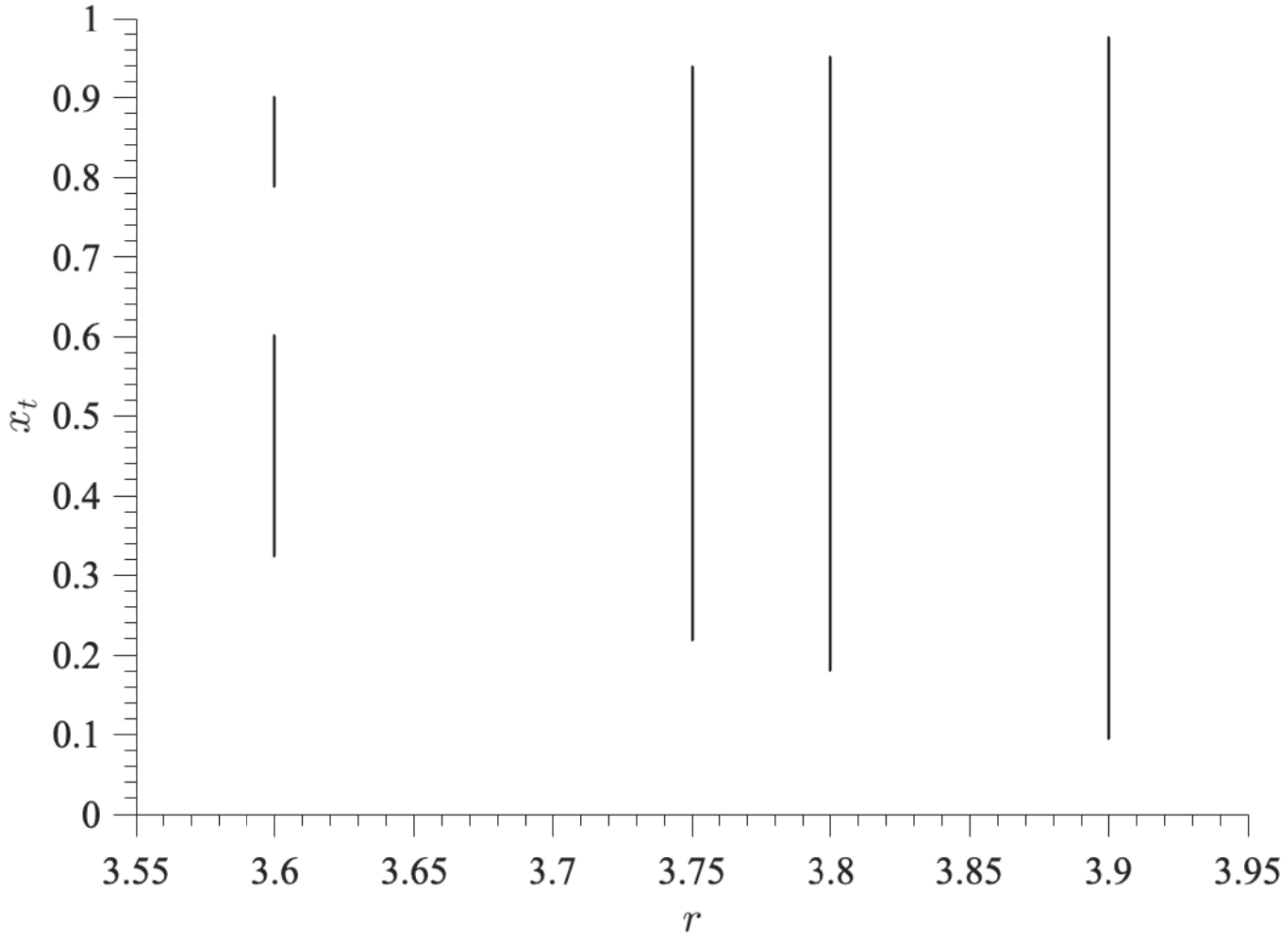
Iterative maps

Bivariate logistic map

$$f(z) = r\,z(1-z)$$

$$\begin{cases} x_{t+1} = & (1-\varepsilon)\, f(x_t) + \varepsilon f(y_t), \\ y_{t+1} = & (1-\varepsilon)\, f(y_t) + \varepsilon f(x_t)\,. \end{cases}$$





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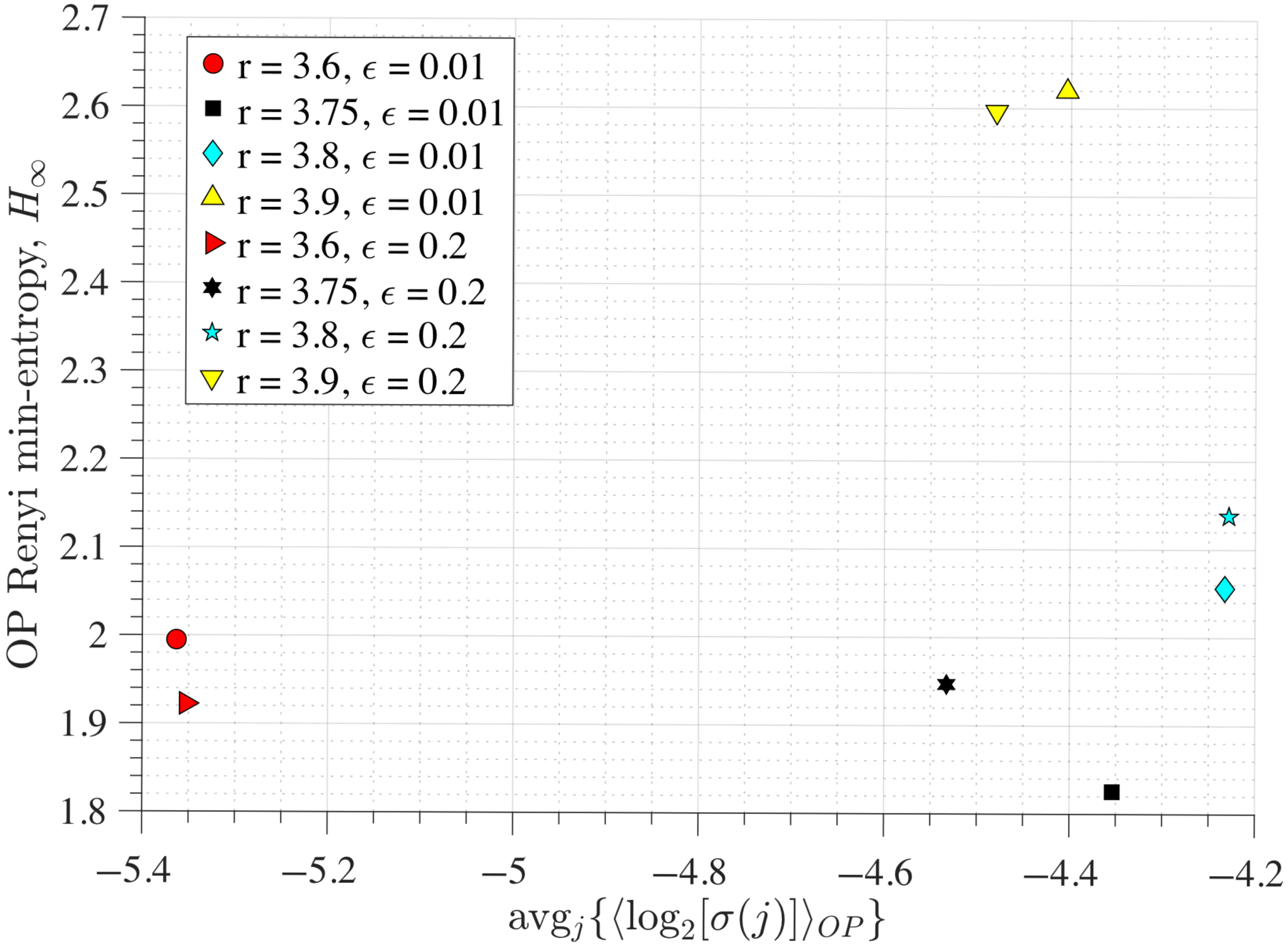
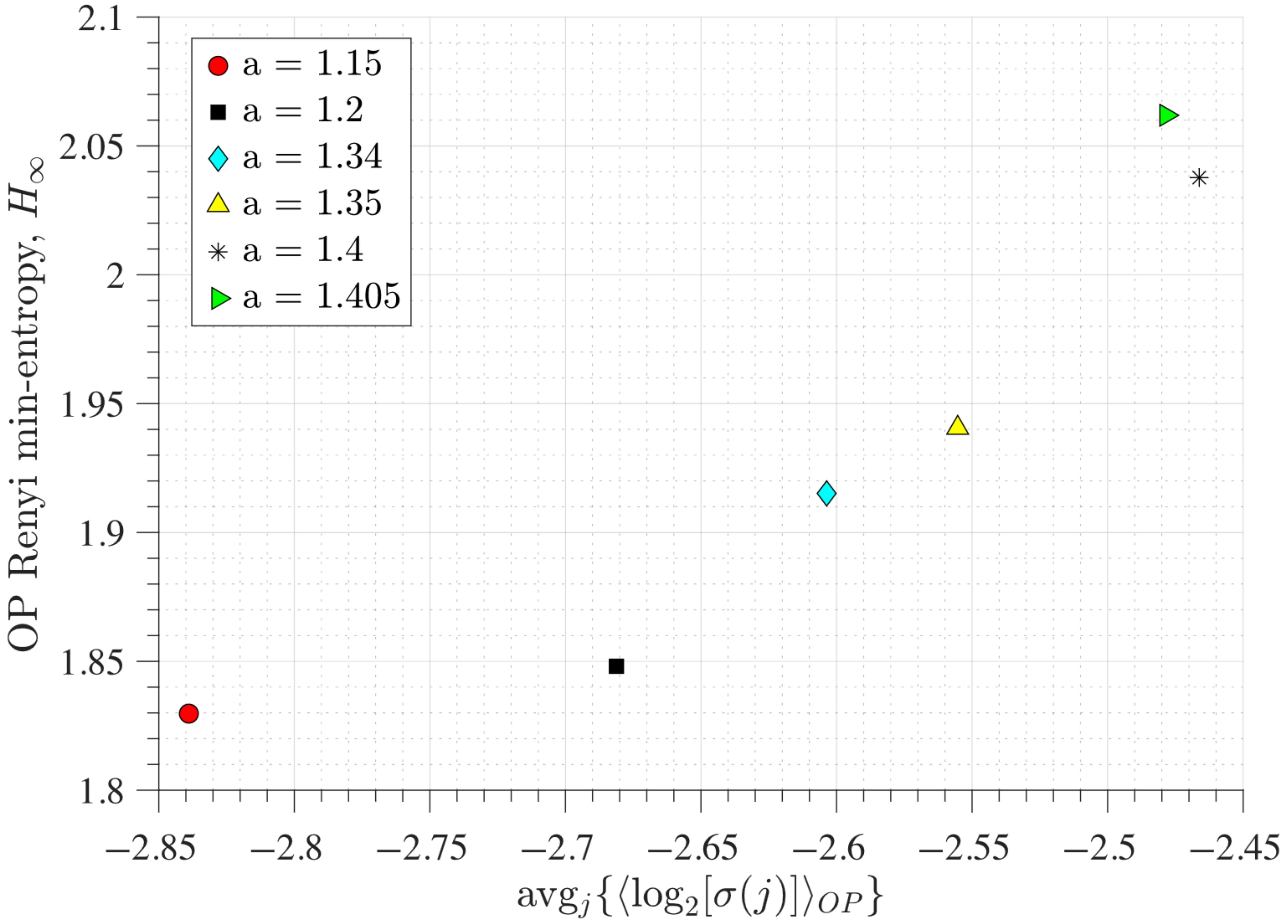


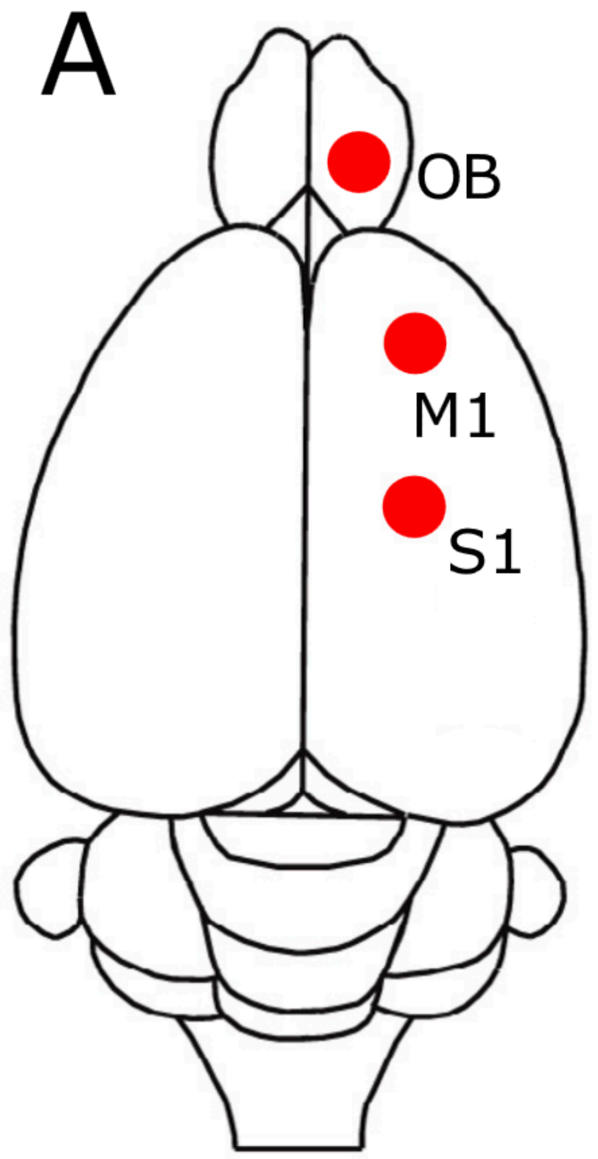
Iterative maps

Hénon map

$$\begin{cases} x_{t+1} = & 1 - a\,x_t^2 + y_t, \\ y_{t+1} = & b\,x_t\,. \end{cases}$$



Empirical data



Olfactory bulb
Primary motor
Somatosensory

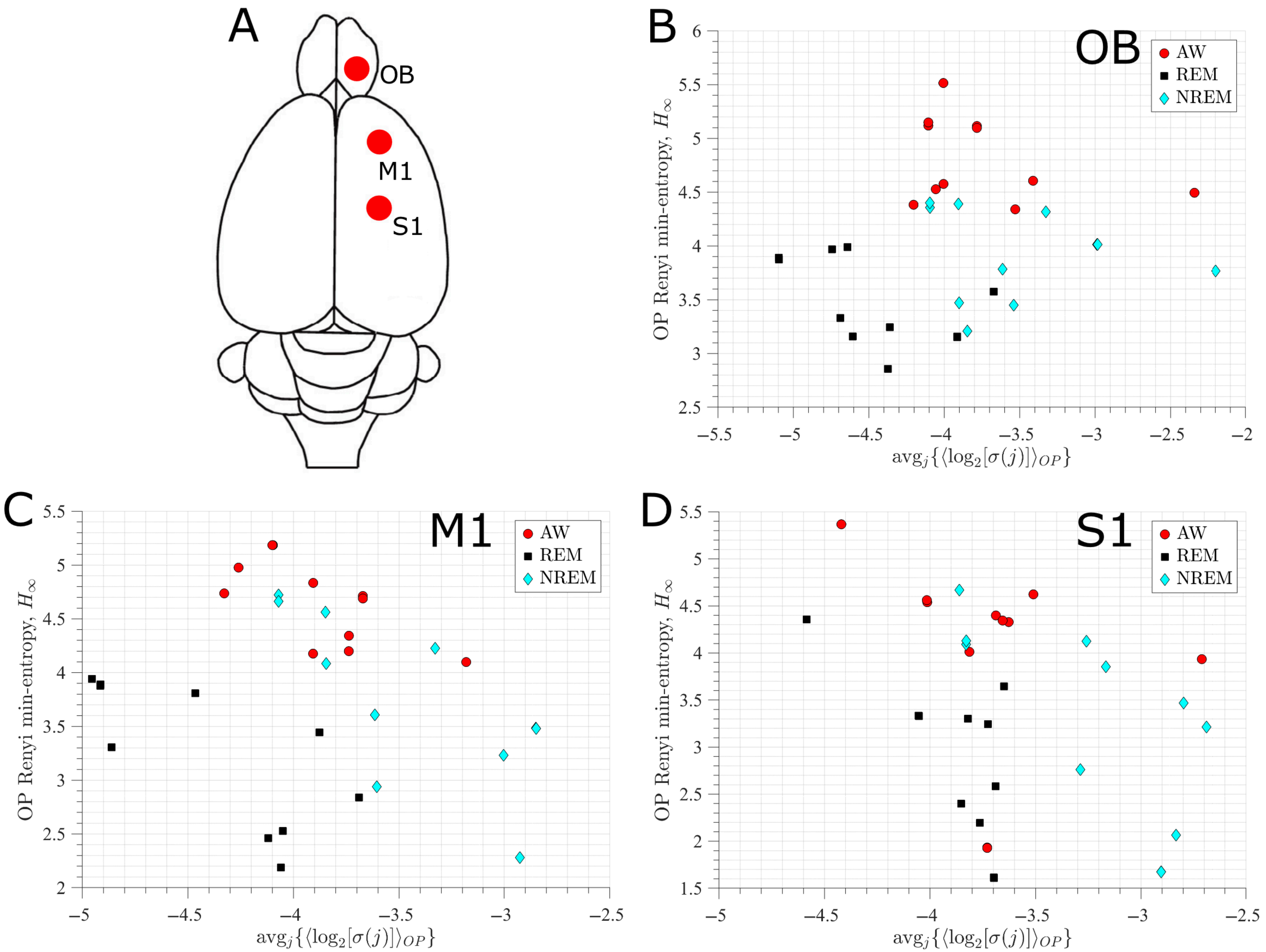
EEG recordings

- 11 rats, freely moving.
- 3 sleep states: active wakefulness, rapid eye movement, Non-REM.
- AW: low-voltage fast waves in M1, a strong theta rhythm in S1. (4–7 Hz), and relatively high electromyographic activity.
- REM: low-voltage fast-frontal waves, a regular theta rhythm in S1, and silent electromyography.
- NREM: high-voltage slow-cortical waves (1–4 Hz), sleep spindles in M1 and S1, and a reduction in electromyographic amplitudes.

Empirical data

EEG recordings

- 3 sleep states: active wakefulness, rapid eye movement, Non-REM.
- AW, REM: Low-voltage
  NREM: High-voltage

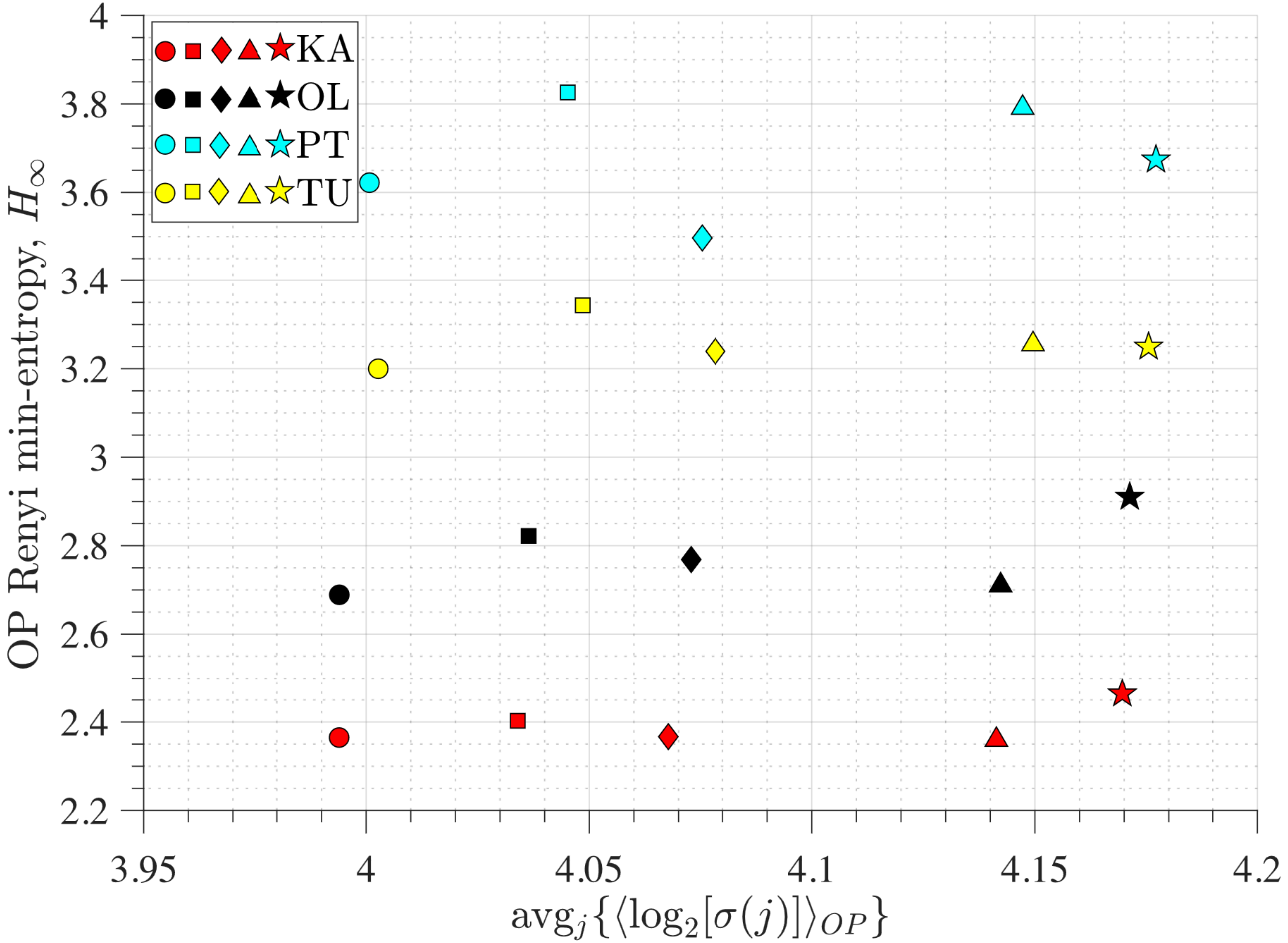


Empirical data

Power grid frequency

- 4 locations in EU grid.
- 5 intervals of 24h.

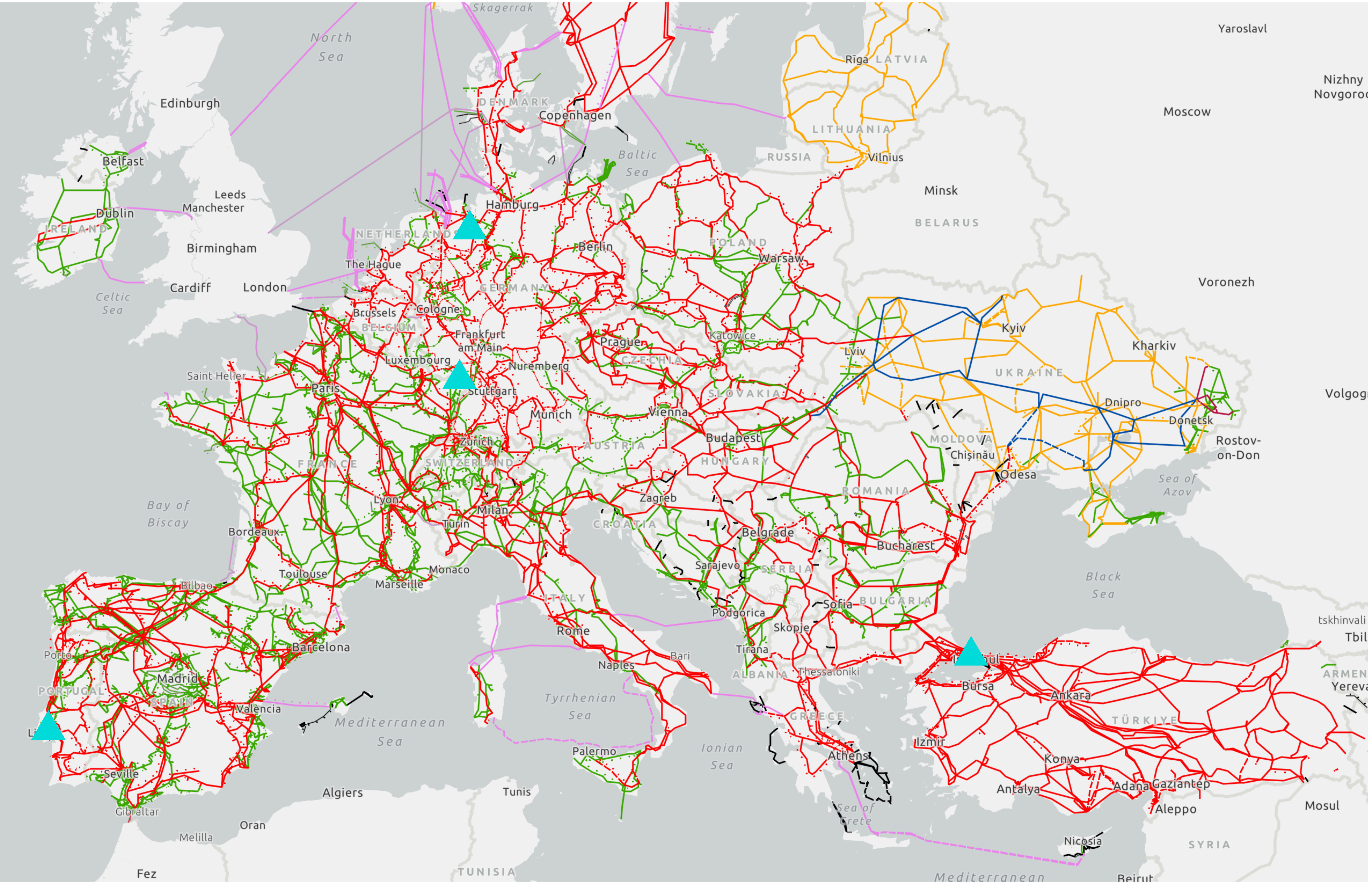
Centrality in the grid
MT, Pagnier, Jacquod
Sci. adv. **5** (11), eaaw8359



Jumar, R.; Maaß, H.; Schäfer, B.; Gorjão, L.R.; Hagenmeyer, V. Database of power grid frequency measurements

Empirical data

Power grid frequency



Less central, more noise

Conclusion

- Keep the magnitude variability in OP analysis
- Could be used to classify time series

Future work

- Include a 3rd dimension using a complexity measure
- Change the standard deviation for the interquartile

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