The Key Player Problem in Realistic Large-Scale Power Grids

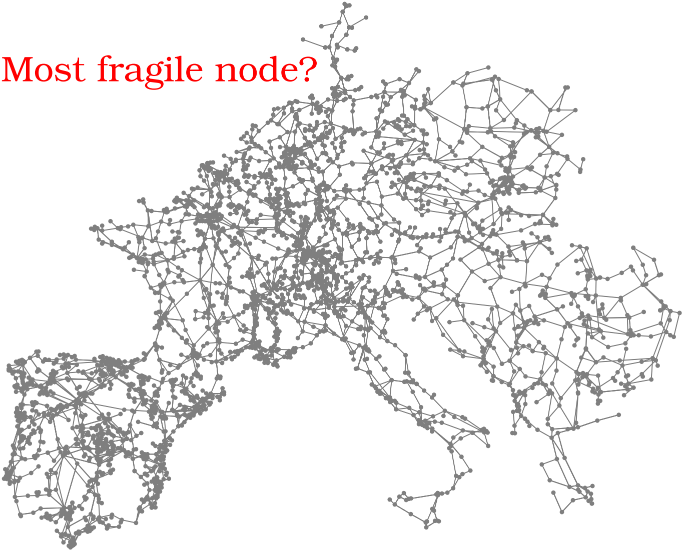
Melvyn Tyloo

University of Applied Sciences of Western Switzerland HES-SO, Sion and Institute of Physics, École Polytechnique Fédérale de Lausanne (EPFL).

September 3, 2019

MT,Pagnier and Jacquod to appear in *Science Advances* (2019), arXiv:1810.09694.

The Key Player Problem



MT,Pagnier and Jacquod to appear in *Science Advances* (2019), arXiv:1810.09694.

Complex Networks Metrics

Centrality Measures (Local)

- Geodesic Distance
- PageRank
- Katz centrality
- Harmonic
- Degree
- Communicability
- Betweenness Centrality
- ...

Graph Measures (Global)

- Average Geodesic Distance
- Degree Heterogeneity
- Average Degree
- Degree Distribution: Scale-Free...
- Clustering Coefficient
- A,Di-ssortativity
- ...

P. Boldi and S. Vigna, Internet Mathematics **10**, 222 (2014).

Complex Networks Metrics

Centrality Measures (Local)

- Geodesic Distance
- PageRank
- Katz centrality
- Harmonic
- Degree
- Communicability
- Betweenness Centrality
- ...

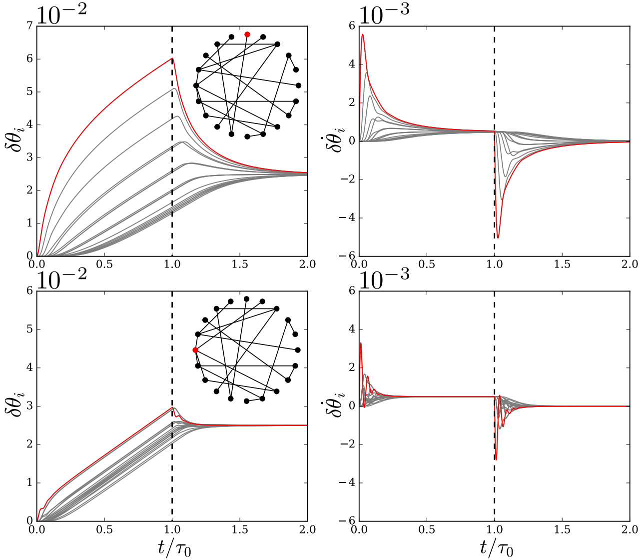
Graph Measures (Global)

- Average Geodesic Distance
- Degree Heterogeneity
- Average Degree
- Degree Distribution: Scale-Free...
- Clustering Coefficient
- A,Di-ssortativity
- ...

Not related to particular network dynamics... can be misleading.

P. Boldi and S. Vigna, Internet Mathematics **10**, 222 (2014).
P. Hines, E. Cortilla-Sanchez and S. Blumsack, Chaos **22**, 033122 (2010).

Coupled Dynamical Systems: Example



MT and Jacquod to appear in *Phys. Rev. E* (2019), arXiv:1905.03582.

Swing Equations in the lossless line limit (second-order Kuramoto):

$$m_i\ddot{\theta}_i + d_i\dot{\theta}_i = P_i - \sum_j b_{ij}\sin(\theta_i - \theta_j) \ \ , \ i = 1, ..., n.$$

$b_{ij} = b_{ji} \geq 0$.

Steady-state solutions: Synchronous state $\{\theta_i^{(0)}\}$ such that:

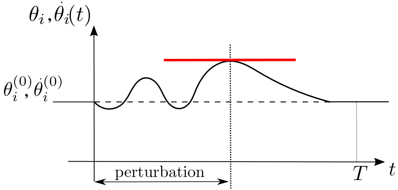
$$P_i = \sum_j b_{ij}\sin(\theta_i^{(0)} - \theta_j^{(0)}) \ \ , \ i = 1, ..., n.$$

$\sum_i P_i = 0.$

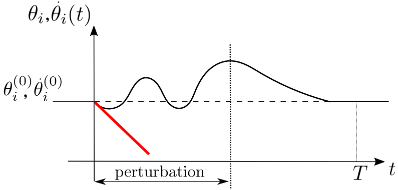
Perturbations: $P_i \rightarrow P_i^{(0)} + \delta P_i(t).$

Quantifying Robustness

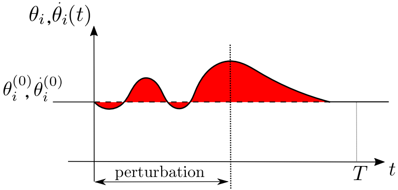
- Maximum of the response, $\max_t(\theta_i)$.



- Rate of change of frequency (RoCoF), $\ddot{\theta}_i$.

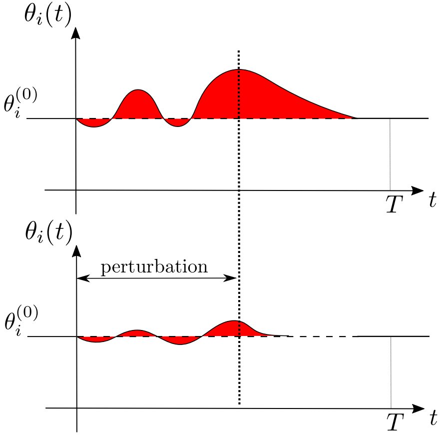


- Performance measure (quadratic integrals over the transient).



Quantifying Robustness

Performance measures :



$$\mathcal{P}_1(T) = \sum_i \int_0^T |\theta_i(t) - \theta_i^{(0)}|^2 \mathrm{d}t \,,$$

$$\mathcal{P}_2(T) = \sum_i \int_0^T |\dot\theta_i(t) - \dot\theta_i^{(0)}|^2 \mathrm{d}t \,.$$

$$\mathcal{P}_{1,2}^\infty = \mathcal{P}_{1,2}(T \to \infty) \,.$$

Noisy disturbances $\to$ divide by T.

Perturbations : $P_i \to P_i^{(0)} + \delta P_i(t)$.

Response to Perturbations: Linearization

Linear response: Perturbation of the natural frequencies.

- $P_i(t) = P_i^{(0)} + \delta P_i(t) \rightarrow \theta_i(t) = \theta_i^{(0)} + \delta\theta_i(t)$:

$$m\delta\ddot{\boldsymbol{\theta}}(t) + d\delta\dot{\boldsymbol{\theta}}(t) = \delta\boldsymbol{P}(t) - \mathbb{L}(\{\theta_i^{(0)}\})\delta\boldsymbol{\theta}(t) \ ,$$

$\mathbb{L}(\{\theta_i^{(0)}\})$: the weighted Laplacian matrix,

$$\mathbb{L}_{ij}(\{\theta_i^{(0)}\}) = \begin{cases} -b_{ij}\cos(\theta_i^{(0)} - \theta_j^{(0)}) \, , & i \neq j \, , \\ \sum_k b_{ik}\cos(\theta_i^{(0)} - \theta_k^{(0)}) \, , & i = j \, . \end{cases}$$

Linear response: Perturbation of the natural frequencies.

- $P_i(t) = P_i^{(0)} + \delta P_i(t) \rightarrow \theta_i(t) = \theta_i^{(0)} + \delta\theta_i(t)$:

$$m\delta\ddot{\boldsymbol{\theta}}(t) + d\delta\dot{\boldsymbol{\theta}}(t) = \delta\boldsymbol{P}(t) - \mathbb{L}(\{\theta_i^{(0)}\})\delta\boldsymbol{\theta}(t) \ ,$$

$\mathbb{L}(\{\theta_i^{(0)}\})$: the weighted Laplacian matrix,

$$\mathbb{L}_{ij}(\{\theta_i^{(0)}\}) = \begin{cases} -b_{ij}\cos(\theta_i^{(0)} - \theta_j^{(0)}) \, , & i \neq j \, , \\ \sum_k b_{ik}\cos(\theta_i^{(0)} - \theta_k^{(0)}) \, , & i = j \, . \end{cases}$$

Topology $\rightarrow b_{ij}$.
Steady state $\rightarrow \{\theta_i^{(0)}\}$.

Linear response

$$m\delta\ddot{\boldsymbol{\theta}}(t) + d\delta\dot{\boldsymbol{\theta}}(t) = \delta\boldsymbol{P}(t) - \mathbb{L}(\{\theta_i^{(0)}\})\delta\boldsymbol{\theta}(t)\ ,$$

Expanding on the eigenvectors $\mathbf{u}_\alpha$ of $\mathbb{L}$, we have $\delta\boldsymbol{\theta}(t) = \sum_\alpha c_\alpha(t)\mathbf{u}_\alpha$.

$$c_\alpha(t) = m^{-1}\, e^{-(\gamma+\Gamma_\alpha)t/2} \int_0^t e^{\Gamma_\alpha t_1} \int_0^{t_1} \delta\boldsymbol{P}(t_2)\cdot\mathbf{u}_\alpha\, e^{(\gamma-\Gamma_\alpha)t_2/2}\,\mathrm{d}t_2\mathrm{d}t_1\ ,$$

$\gamma = d/m$ and $\Gamma_\alpha = \sqrt{\gamma^2 - 4\lambda_\alpha/m}$. $\rightarrow \mathcal{P}_1$, $\mathcal{P}_2$ for specific perturbations,

$$\begin{aligned}
\mathcal{P}_1^\infty &= \sum_{\alpha\geq 2}\int_0^\infty c_\alpha^2(t)\mathrm{d}t\ ,\\
\mathcal{P}_2^\infty &= \sum_{\alpha\geq 2}\int_0^\infty \dot{c}_\alpha^2(t)\mathrm{d}t\ .
\end{aligned}$$

Response to Perturbations: Time Scales

Intrinsic Time Scales

- Individual elements: m/d.
- Network relaxation: d/λ_α with $\{\lambda_\alpha\}$ the eigenvalues of $\mathbb{L}$.

Perturbation Time Scale

- Correlation time of the external perturbation $\delta\boldsymbol{P}(t)$.

Quench perturbations

- $\delta P_i(t) = \delta P_{0i}\Theta(t)\Theta(\tau_0 - t)$.

Quench duration $\rightarrow \tau_0$.

Noisy time correlated perturbations

- $\langle \delta P_i(t) P_j(t') \rangle = \delta P_{0i}^2 \delta_{ij} \exp[-|t-t'|/\tau_0]$.

Correlation time $\rightarrow \tau_0$.

MT,Pagnier and Jacquod to appear in *Science Advances* (2019), arXiv:1810.09694.
MT and Jacquod to appear in *Phys. Rev. E* (2019), arXiv:1905.03582.

Performance Measures for Noisy Perturbations

$$\mathcal{P}_1^\infty = \delta P_0^2 \sum_{\alpha\geq 2} \frac{u_{\alpha,k}^2(\tau_0 + m/d)}{\lambda_\alpha(\lambda_\alpha\tau_0 + d + m/\tau_0)} \,, \qquad (1)$$

$$\mathcal{P}_2^\infty = \delta P_0^2 \sum_{\alpha\geq 2} \frac{u_{\alpha,k}^2}{d(\lambda_\alpha\tau_0 + d + m/\tau_0)} \,. \qquad (2)$$

MT,Pagnier and Jacquod to appear in *Science Advances* (2019), arXiv:1810.09694.

Performance Measures Asymptotics

Short time correlation: $\tau_0 \ll d/\lambda_\alpha, m/d$

$$\mathcal{P}_1^\infty \simeq \frac{\tau_0}{d} \sum_{\alpha \geq 2} \frac{\delta P_0^2 u_{\alpha,k}^2}{\lambda_\alpha} \, , \tag{3}$$

$$\mathcal{P}_2^\infty \simeq \frac{\tau_0}{dm} \sum_{\alpha \geq 2} \delta P_0^2 u_{\alpha,k}^2 \, . \tag{4}$$

Long time correlation: $\tau_0 \gg d/\lambda_\alpha, m/d$

$$\mathcal{P}_1^\infty \simeq \delta P_0^2 \sum_{\alpha \geq 2} \frac{u_{\alpha,k}^2}{\lambda_\alpha^2} \, , \tag{5}$$

$$\mathcal{P}_2^\infty \simeq \delta P_0^2 \frac{1}{d\tau_0} \sum_{\alpha \geq 2} \frac{u_{\alpha,k}^2}{\lambda_\alpha} \, . \tag{6}$$

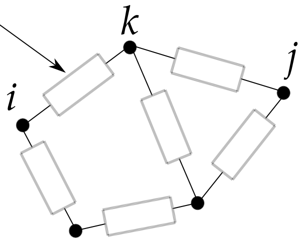
MT,Pagnier and Jacquod to appear in *Science Advances* (2019), arXiv:1810.09694.

Resistance Distance

$$\Omega_{ij} \;=\; \mathbb{L}^\dagger_{ii} + \mathbb{L}^\dagger_{jj} - \mathbb{L}^\dagger_{ij} - \mathbb{L}^\dagger_{ji} = \sum_{\alpha \geq 2} \frac{(u_{\alpha,i} - u_{\alpha,j})^2}{\lambda_\alpha} \,.$$

$\mathbb{L}^\dagger$: pseudo inverse of $\mathbb{L}$ (because of $\lambda_1 = 0$).

$R_{ik} = [b_{ik}\cos(\theta_i^{(0)} - \theta_k^{(0)})]^{-1}$

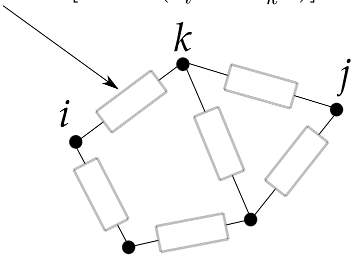


Klein and Randić, *J. Math. Chem.* **12**, 81 (1993).

Kirchhoff Index

$$Kf_1 = \sum_{i<j} \Omega_{ij} = n \sum_{\alpha \geq 2} \lambda_\alpha^{-1} \, .$$

$$R_{ik} = [b_{ik} \cos(\theta_i^{(0)} - \theta_k^{(0)})]^{-1}$$

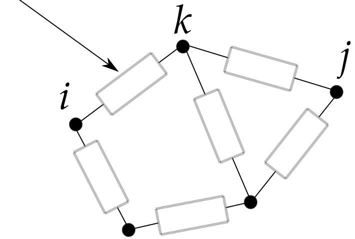


Klein and Randić, *J. Math. Chem.* **12**, 81 (1993).

Resistance Centrality

$$C_1(k) = \left[n^{-1} \sum_j \Omega_{kj}\right]^{-1} = \left[\sum_{\alpha \geq 2} \frac{u_{\alpha,k}^2}{\lambda_\alpha} + n^{-2} Kf_1\right]^{-1} .$$

$$R_{ik} = [b_{ik} \cos(\theta_i^{(0)} - \theta_k^{(0)})]^{-1}$$



Resistance Distances, $Kf_p's$ and C_p's

Generalized Resistance Distances

$$\Omega_{ij}^{(p)} = \mathbb{L}'^{\dagger}_{ii} + \mathbb{L}'^{\dagger}_{jj} - \mathbb{L}'^{\dagger}_{ij} - \mathbb{L}'^{\dagger}_{ji} = \sum_{\alpha\geq 2} \frac{(u_{\alpha,i} - u_{\alpha,j})^2}{\lambda_\alpha^p} \,,$$

$$\mathbb{L}' = \mathbb{L}^p \,.$$

Generalized Kirchhoff Indices

$$Kf_p = \sum_{i<j} \Omega_{ij}^{(p)} = n \sum_{\alpha\geq 2} \lambda_\alpha^{-p} \,.$$

Generalized Resistance Centralities

$$C_p(k) = \left[n^{-1} \sum_j \Omega_{kj}^{(p)} \right]^{-1} = \left[\sum_{\alpha\geq 2} \frac{u_{\alpha,k}^2}{\lambda_\alpha^p} + n^{-2} Kf_p \right]^{-1} .$$

MT,Coletta and Jacquod, *Phys. Rev. Lett.* **120**, 084101 (2018).

MT,Pagnier and Jacquod to appear in *Science Advances* (2019), arXiv:1810.09694.

MT and Jacquod to appear in *Phys. Rev. E* (2019), arXiv:1905.03582.

Specific Local Vulnerabilities $\to C_p$'s

Local Vulnerability

Perturbing a specific node k i.e. $\delta P_{0i} = \delta_{ik}\delta P_0$,

$\tau_0 \ll m/d, (\gamma \pm \Gamma_\alpha)^{-1}$

$$\begin{aligned}
\mathcal{P}_1^\infty(k) &= \frac{\delta P_0^2 \tau_0^2}{2d}[C_1^{-1}(k) - n^{-2}Kf_1]\,,\\
\mathcal{P}_2^\infty(k) &= \frac{\delta P_0^2 \tau_0^2}{2md}\frac{(n-1)}{n}\,,
\end{aligned}$$

$\tau_0 \gg m/d, (\gamma \pm \Gamma_\alpha)^{-1}$

$$\begin{aligned}
\mathcal{P}_1^\infty(k) &= \delta P_0^2 \tau_0 [C_2^{-1}(k) - n^{-2}Kf_2]\,,\\
\mathcal{P}_2^\infty(k) &= \frac{\delta P_0^2}{d}[C_1^{-1}(k) - n^{-2}Kf_1]\,.
\end{aligned}$$

MT,Pagnier and Jacquod to appear in *Science Advances* (2019), arXiv:1810.09694.

Local Vulnerability
Perturbing a specific node k i.e. $\delta P_{0i} = \delta_{ik}\delta P_0$,
$\tau_0 \ll m/d$, $(\gamma \pm \Gamma_\alpha)^{-1}$

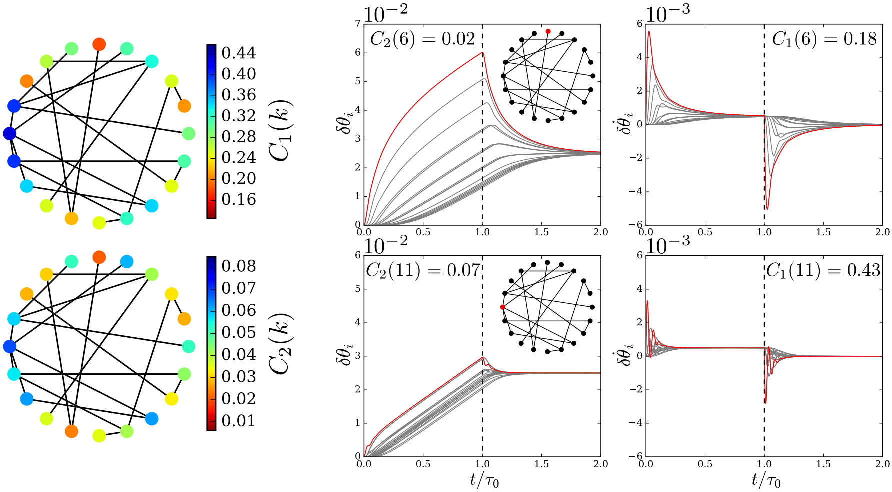
$$\begin{aligned}
\mathcal{P}_1^\infty(k) &= \frac{\delta P_0^2 \tau_0^2}{2d}[C_1^{-1}(k) - n^{-2}Kf_1]\,,\\
\mathcal{P}_2^\infty(k) &= \frac{\delta P_0^2 \tau_0^2}{2md}\frac{(n-1)}{n}\,,
\end{aligned}$$

$\tau_0 \gg m/d$, $(\gamma \pm \Gamma_\alpha)^{-1}$

$$\begin{aligned}
\mathcal{P}_1^\infty(k) &= \delta P_0^2 \tau_0 [C_2^{-1}(k) - n^{-2}Kf_2]\,,\\
\mathcal{P}_2^\infty(k) &= \frac{\delta P_0^2}{d}[C_1^{-1}(k) - n^{-2}Kf_1]\,.
\end{aligned}$$

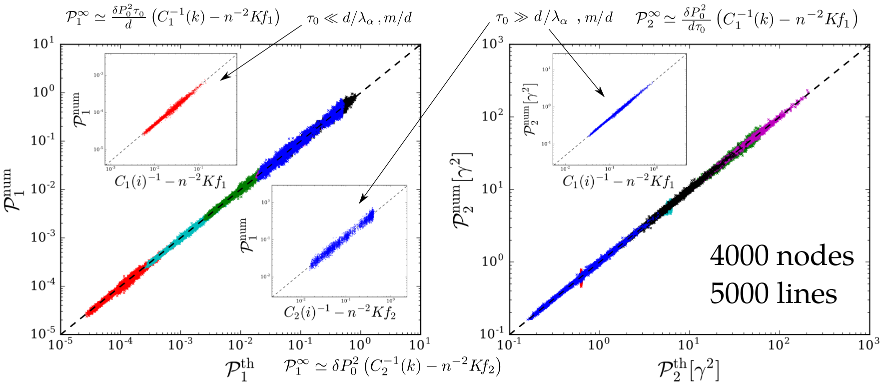
MT,Pagnier and Jacquod to appear in *Science Advances* (2019), arXiv:1810.09694.

Specific Local Vulnerabilities and C_p's: Numerics



MT and Jacquod to appear in *Phys. Rev. E* (2019), arXiv:1905.03582.

Specific Local Vulnerabilities and C_p's: Numerics



MT,Pagnier and Jacquod to appear in *Science Advances* (2019), arXiv:1810.09694.

Physical Realization : European Electrical Grid

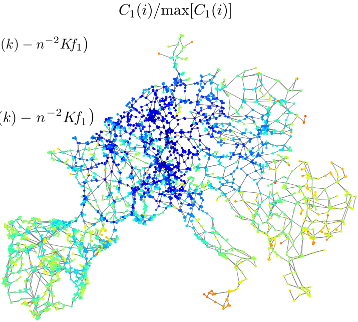
$\tau_0 \ll d/\lambda_\alpha\,, m/d$

$\mathcal{P}_1^\infty \simeq \frac{\delta P_0^2 \tau_0}{d}\left(C_1^{-1}(k) - n^{-2}Kf_1\right)$

$\mathcal{P}_2^\infty \simeq \frac{\delta P_0^2 \tau_0}{dm}\frac{(n-1)}{n}$

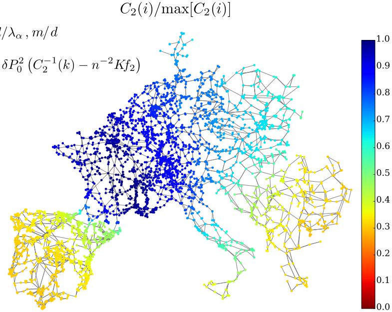
$\tau_0 \gg d/\lambda_\alpha \;\;, m/d$

$\mathcal{P}_2^\infty \simeq \frac{\delta P_0^2}{d\tau_0}\left(C_1^{-1}(k) - n^{-2}Kf_1\right)$



$\tau_0 \gg d/\lambda_\alpha\,, m/d$

$\mathcal{P}_1^\infty \simeq \delta P_0^2\left(C_2^{-1}(k) - n^{-2}Kf_2\right)$



MT,Pagnier and Jacquod to appear in *Science Advances* (2019), arXiv:1810.09694.

Ranking of Vulnerabilities

Ranking of the nodes

$\mathrm{LRank}_{1,2}$: Based on $C_{1,2}$ related to

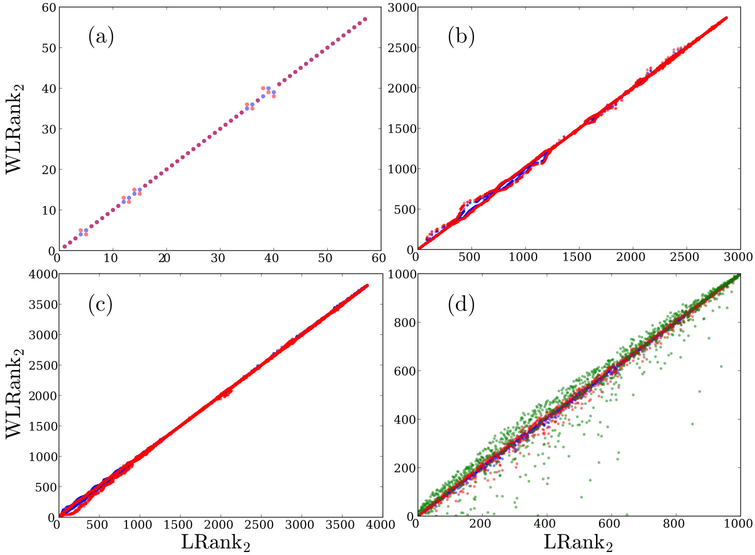
$$\mathbb{L}_{ij}(\{\theta_i^{(0)}\}) = \begin{cases} -b_{ij}\,, & i \neq j\,, \\ \sum_k b_{ik}\,, & i = j\,. \end{cases}$$

$\mathrm{WLRank}_{1,2}$: Based on $C_{1,2}$ related to

$$\mathbb{L}_{ij}(\{\theta_i^{(0)}\}) = \begin{cases} -b_{ij}\cos(\theta_i^{(0)} - \theta_j^{(0)})\,, & i \neq j\,, \\ \sum_k b_{ik}\cos(\theta_i^{(0)} - \theta_k^{(0)})\,, & i = j\,. \end{cases}$$

MT,Pagnier and Jacquod to appear in *Science Advances*, arXiv:1810.09694.

Ranking of Vulnerabilities



MT,Pagnier and Jacquod to appear in *Science Advances*, arXiv:1810.09694.

Local Vulnerabilities

- Most vulnerable nodes $\rightarrow C_1$ or C_2 (C_p) depending on τ_0 and on the dynamical variable of interest.
  $\rightarrow p$ depends on which performance measures you are interested in and on the correlation time of the perturbation.
- Rank the nodes: independent of the operational/synchronous state if $|\Delta\theta| < 30^\circ \rightarrow \mathrm{LRank} \cong \mathrm{WLRank}$.

Inertia

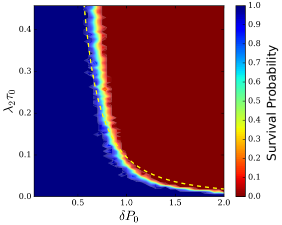
- No effect on performance measures in both asymptotics in τ_0 except for frequencies and short τ_0.

MT,Pagnier and Jacquod to appear in *Science Advances*, arXiv:1810.09694.

Conclusion

Response to Large Perturbations

→ *Noise-induced desynchronization and stochastic escape from equilibrium in complex networks*, MT,Delabays and Jacquod *Phys. Rev. E* **99**, 062213 (2019).



Line perturbations

→ *Rate of change of frequency under line contingencies in high voltage electric power networks with uncertainties*, Delabays, MT and Jacquod, arXiv:1906.05698.

HES-SO Valais-Wallis

Physical Realization : European Electrical Grid

$\tau_0 \ll d/\lambda_\alpha \,, m/d$

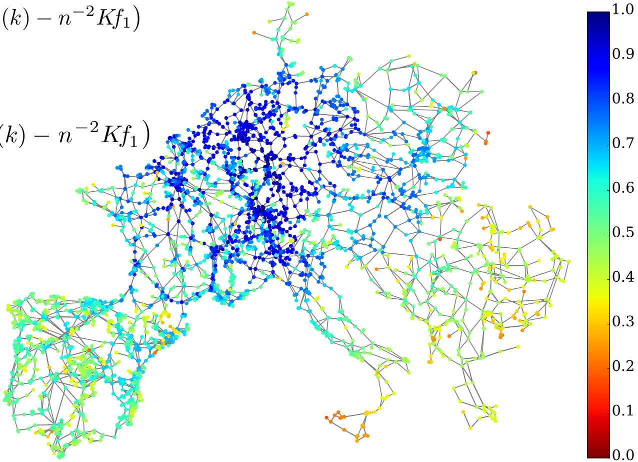
$\mathcal{P}_1^\infty \simeq \frac{\delta P_0^2 \tau_0}{d} \left(C_1^{-1}(k) - n^{-2} K f_1\right)$

$\mathcal{P}_2^\infty \simeq \frac{\delta P_0^2 \tau_0}{dm} \frac{(n-1)}{n}$

$\tau_0 \gg d/\lambda_\alpha \;\; , m/d$

$\mathcal{P}_2^\infty \simeq \frac{\delta P_0^2}{d\tau_0} \left(C_1^{-1}(k) - n^{-2} K f_1\right)$

$C_1(i)/\max[C_1(i)]$

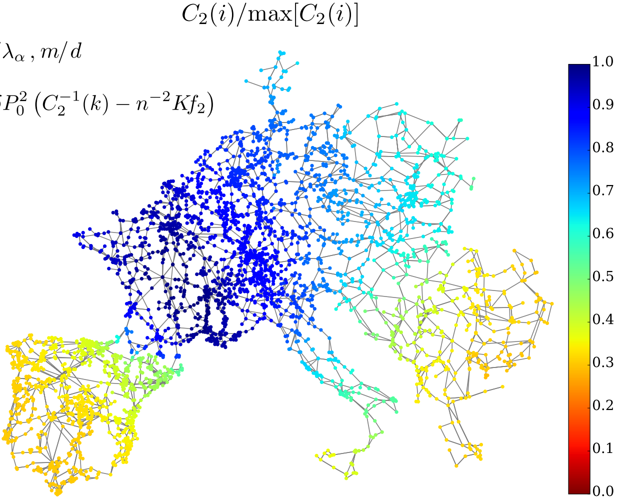


MT,Pagnier and Jacquod to appear in *Science Advances*, arXiv:1810.09694.

Physical Realization : European Electrical Grid

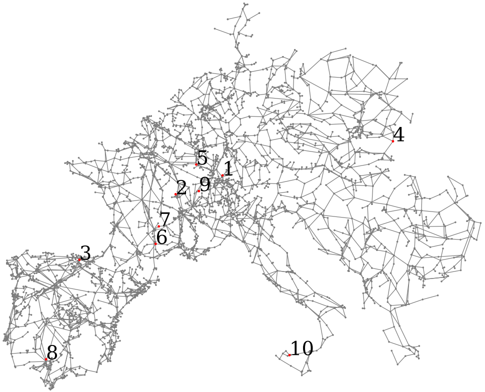
$\tau_0 \gg d/\lambda_\alpha\,, m/d$

$\mathcal{P}_1^\infty \simeq \delta P_0^2 \left(C_2^{-1}(k) - n^{-2} K f_2\right)$



MT,Pagnier and Jacquod to appear in *Science Advances*, arXiv:1810.09694.

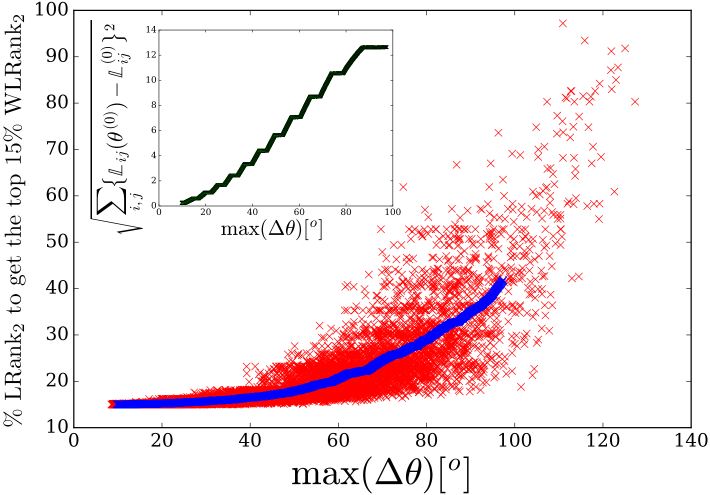
Physical Realization : European Electrical Grid



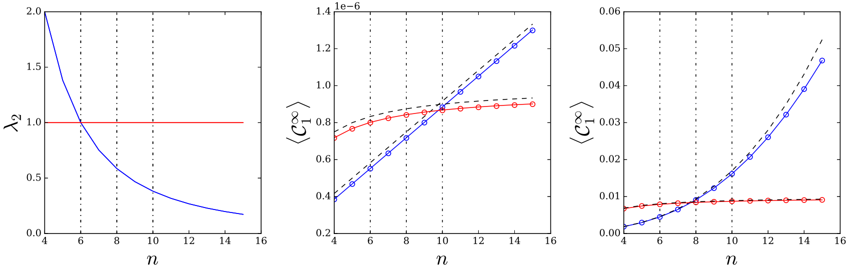
node #	$C_{\rm geo}$	Degree	PageRank	C_1	C_2	$\mathcal{P}_1^{\rm num}$	$\mathcal{P}_2^{\rm num}$ $[\gamma^2]$
1	7.84	4	3024	31.86	5.18	0.047	0.035
2	6.8	1	2716	22.45	5.68	0.021	0.118
3	5.56	10	896	22.45	2.33	0.32	0.116
4	4.79	3	1597	21.74	3.79	0.126	0.127
5	7.08	1	1462	21.74	5.34	0.026	0.125
6	4.38	6	2945	21.69	5.65	0.023	0.129
7	5.11	2	16	19.4	5.89	0.016	0.164
8	4.15	6	756	19.38	1.83	0.453	0.172
9	5.06	1	1715	10.2	5.2	0.047	0.449
10	2.72	4	167	7.49	2.17	0.335	0.64

MT,Pagnier and Jacquod to appear in *Science Advances*, arXiv:1810.09694.

Ranking of Vulnerabilities



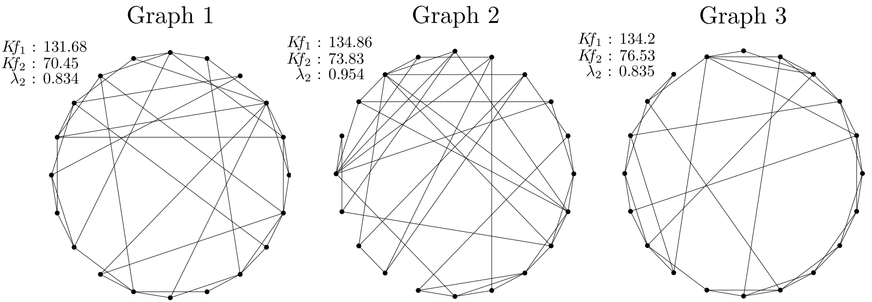
MT,Pagnier and Jacquod to appear in *Science Advances*, arXiv:1810.09694.



blue : cycle graph
red : star graph

MT,Coletta and Jacquod, *Phys. Rev. Lett.* **120**, 084101 (2018)

Supplemental Material



MT,Coletta and Jacquod, *Phys. Rev. Lett.* **120**, 084101 (2018)