Fluctuations in Layered Complex Networks

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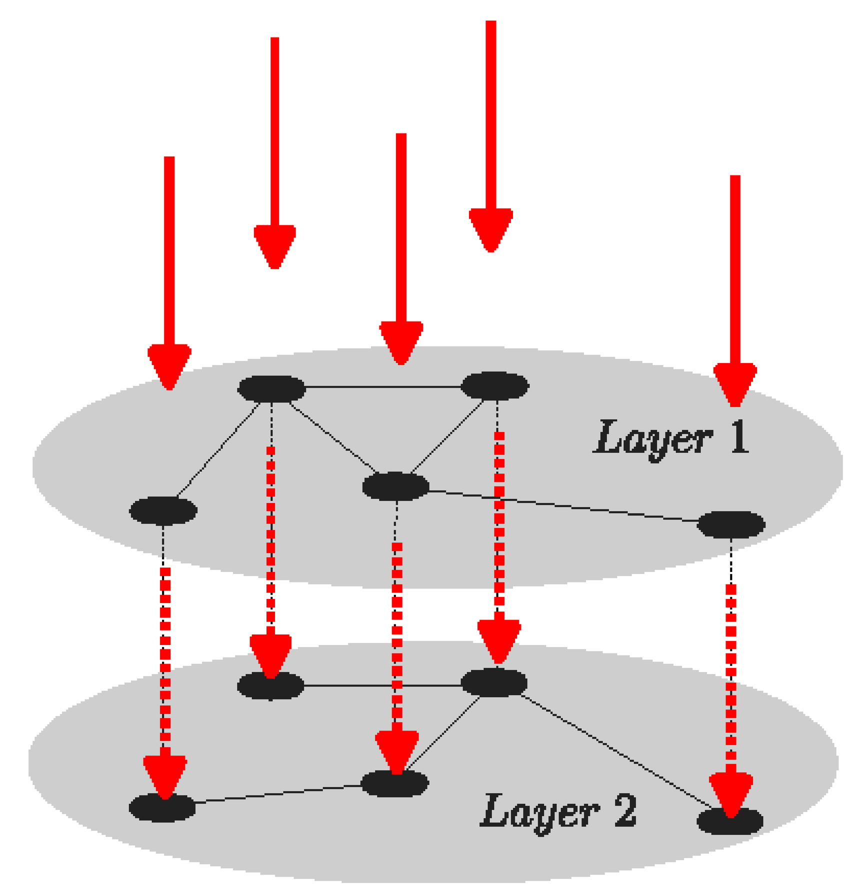
Layered complex systems

- **More accurate description of coupled systems.**
- **Interaction between dynamical systems.**

Goals

- **Predict the propagation of fluctuations coming from one layer to the others.**
- **Investigate the effect of system specific correlation in the noise.**
- **Prevent system failures**



Two layers networked system

Diffusively coupled systems inter-connected with a directed coupling.

$$\dot{x}_i = -\sum_{j=1}^{n} \mathbb{L}_{ij}^{(1)} x_j + \eta_i \quad i = 1, \dots n,$$

$$\dot{y}_i = -\sum_{j=1}^{n} \mathbb{L}_{ij}^{(2)} y_j + f_i(\{x_k\},\{y_k\}) \quad i = 1, \dots n$$

Simple choice of coupling function:

$$f_i(\{x_k\},\{y_k\}) = x_i - n^{-1} \sum_j x_j$$

Noise Propagation

Time-evolution for each layer:

$$x_i(t) = \sum_\alpha \mathrm{e}^{-\lambda_\alpha^{(1)} t} \int_0^t \mathrm{e}^{\lambda_\alpha^{(1)} t'} \sum_j \eta_j u_{\alpha,j}^{(1)} \,\mathrm{d}t'\, u_{\alpha,i}^{(1)},$$

$$y_i(t) = \sum_\alpha \mathrm{e}^{-\lambda_\alpha^{(2)} t} \int_0^t \mathrm{e}^{\lambda_\alpha^{(2)} t'} \sum_j x_j u_{\alpha,j}^{(2)} \,\mathrm{d}t'\, u_{\alpha,i}^{(2)}.$$

Variance in each layer:

$$\langle x_i^2 \rangle = \frac{\eta_0^2}{2} \sum_\alpha \frac{u_{\alpha,i}^{(1)\,2}}{\lambda_\alpha^{(1)}},$$

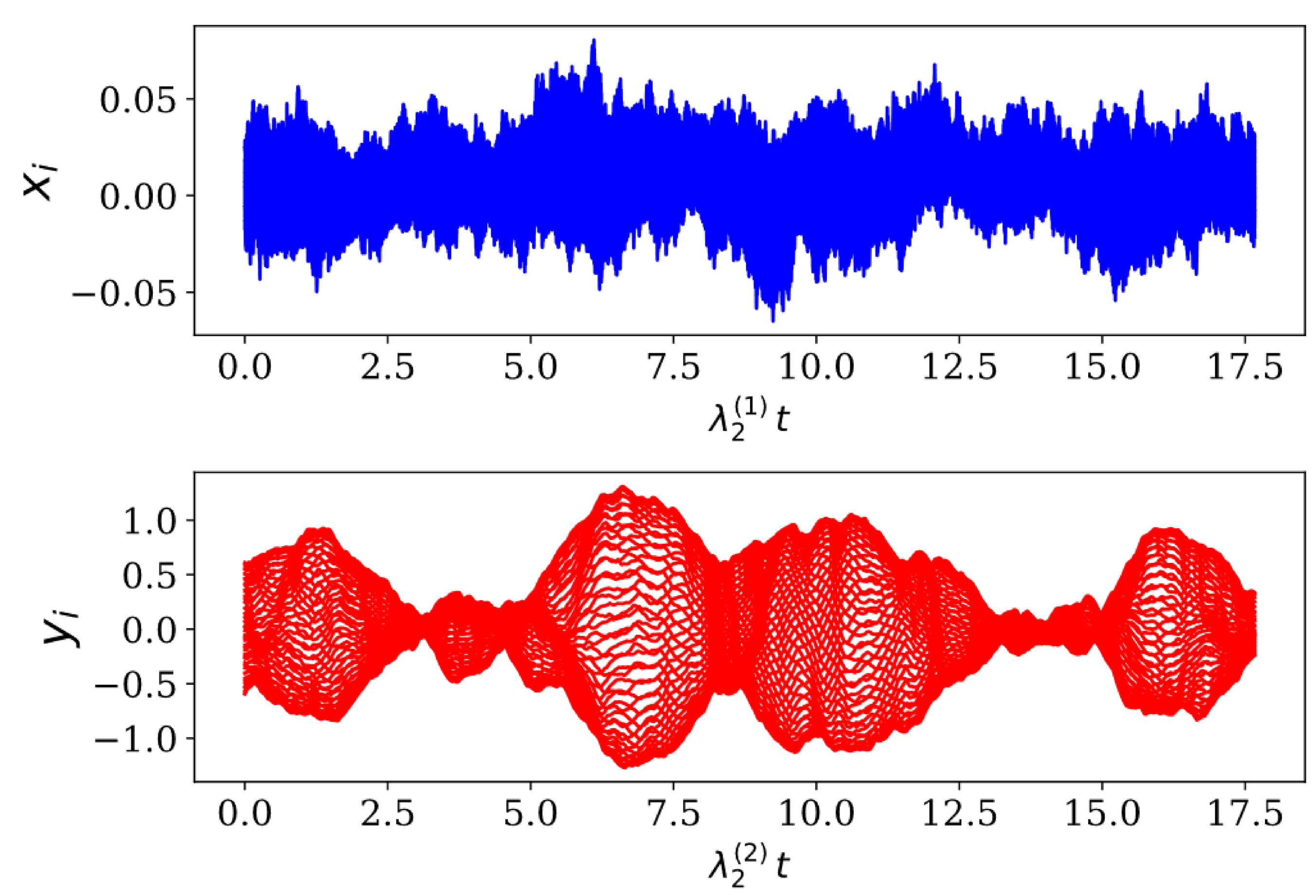
$$\langle y_i^2 \rangle = \frac{\eta_0^2}{2} \sum_{\alpha,\beta,\gamma} \sum_{k,l} \frac{u_{\gamma,k}^{(1)} u_{\gamma,l}^{(1)} u_{\alpha,k}^{(2)} u_{\beta,l}^{(2)} [2\lambda_\gamma^{(1)} + \lambda_\alpha^{(2)} + \lambda_\beta^{(2)}]}{\lambda_\gamma^{(1)} (\lambda_\alpha^{(2)} + \lambda_\beta^{(2)})(\lambda_\gamma^{(1)} + \lambda_\alpha^{(2)})(\lambda_\gamma^{(1)} + \lambda_\beta^{(2)})} u_{\alpha,i}^{(2)} u_{\beta,i}^{(2)}.$$

Same networks:

$$\langle y_i^2 \rangle = \frac{\eta_0^2}{4} \sum_\alpha \frac{u_{\alpha,i}^2}{\lambda_\alpha^3}.$$

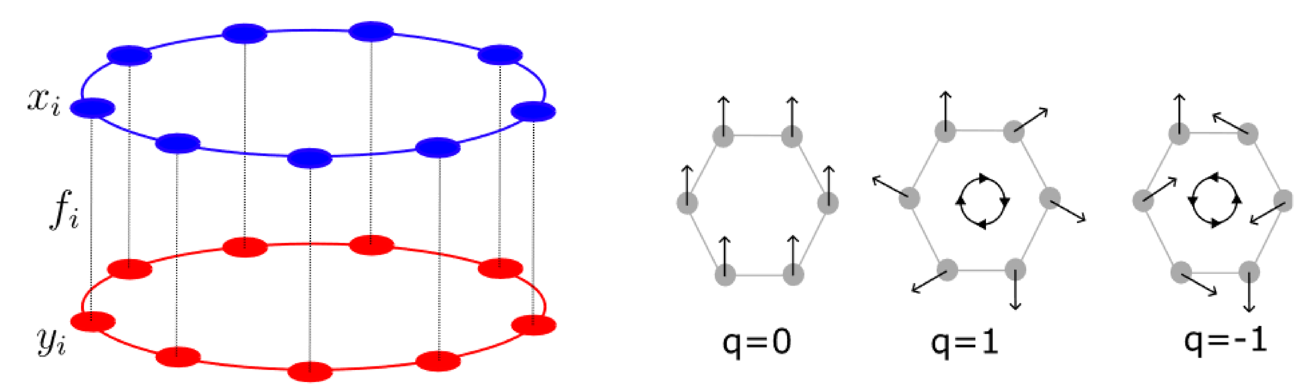
Algebraic connectivity determines whether fluctuations are amplified or reduced!

Cycle networks with small algebraic connectivity

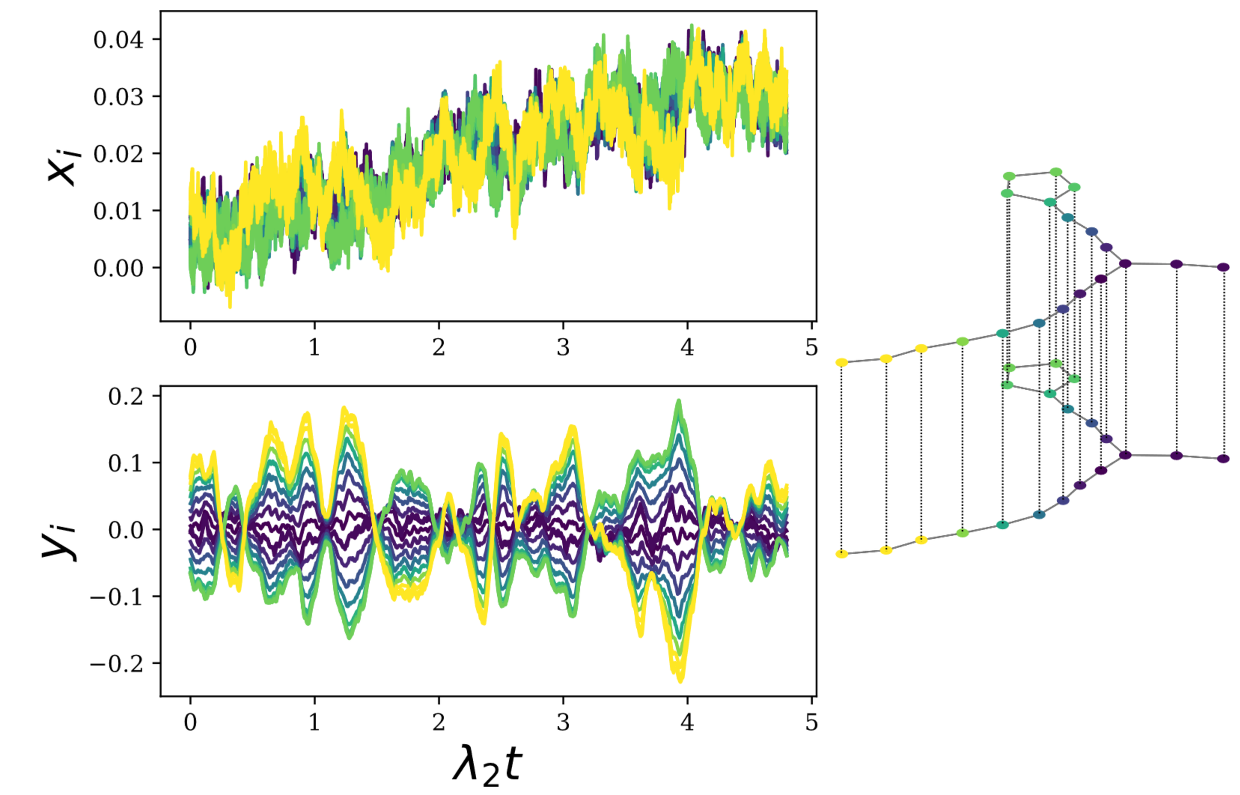


Escape times in nonlinearly coupled systems

Multistability in Kuramoto oscillators:



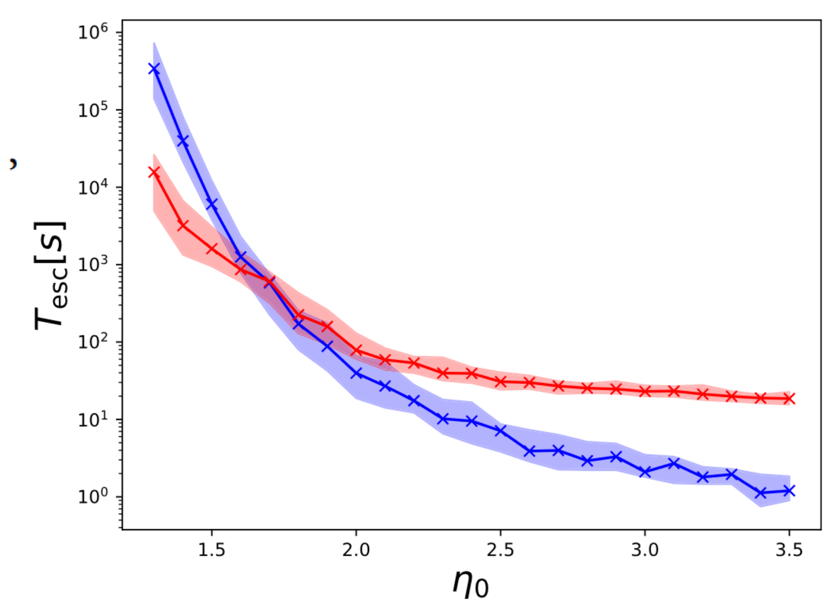
Shape of the noise:



Escape from the initial basin of attraction with rescaled noise:

$$f_i(\{x_k\},\{y_k\}) = d\left(\delta x_i - n^{-1} \sum_j \delta x_j\right),$$

$$d^2 = 2/\textstyle\sum_\alpha \lambda_\alpha^{(1)\,-1}$$



Conclusions

- ✓ **Layered networks: cannot be considered independently.**
- ✓ **More complexity → richer network dynamics ... more vulnerabilities.**

References:

1. M.Tyloo, Journal of Physics: Complexity **3** (3), 03LT01 (2022).
2. M.Tyloo, Chaos **32** (12), 121102 (2022).