Propagation of non-Gaussian voltage angle fluctuations in high-voltage power grids

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High-voltage power grids of the future

• More fluctuations from renewable energy sources, possibly non-Gaussian.
• Less inertia at generator nodes.

Goals

➢ Predict the propagation of fluctuations coming from renewable energy sources.
➢ Clarify the role of damping and inertia.

In the following, we investigate voltage angle fluctuations in realistic high-voltage power grids, induced by renewable energy sources.

Dynamics of AC power grids

Linearized Swing Equations

\[ M \ddot{\delta} + D \dot{\delta} + \delta p(t) - L \dot{\delta}, \]

with \( \delta(t) = \theta^{(0)} + \delta \theta(t) \), \( M = \text{diag}(\{ m_i \}) \) and \( D = \text{diag}(\{ d_i \}) \), and the weighted Laplacian matrix

\[ L_{ij} = \begin{cases} -R_i \cos(\theta_i^{(0)} - \theta_j^{(0)}) & \text{for } i \neq j, \\ \sum_k B_{ik} \cos(\theta_i^{(0)} - \theta_j^{(0)}) & \text{for } i = j, \end{cases} \]

Modal decomposition over the eigenmodes of \( L \)

\[ m \ddot{c}_a + d \dot{c}_a + \lambda_a c_a = \delta p(t) \cdot u_a, \]

\[ c_a(t) = m^{-1} e^{-\gamma \Gamma_\alpha t/2} \int_0^t e^{\gamma \Gamma_\alpha t_2} \delta p(t_1) \cdot u_a \, dt_1 \, dt_2, \]

with \( \Gamma_\alpha = \sqrt{\gamma^2 - 4\lambda_a/m} \) and \( \gamma = d/m \).

Modelling fluctuations of renewable energy sources

Cumulants of time-correlated noise

\[ \langle \delta p_i(t_1) \delta p_j(t_2) \rangle = \sigma^2 \delta_{ij} \delta_{t_1 t_2}, \]

\[ \langle \delta p_i(t_1) \delta p_j(t_2) \delta p_k(t_3) \rangle = a_3 \sigma^4 \Delta_{ijk} \delta_{t_1 t_2}, \]

\[ \langle \delta p_i(t_1) \delta p_j(t_2) \delta p_k(t_3) \delta p_l(t_4) \rangle = a_4 \sigma^4 \Delta_{ijkl} \delta_{t_1 t_2}. \]

Time-scales

• Intrinsic network time-scales < few seconds
  \[ \gamma^{-1} \approx 2.5 \text{s}, T_\alpha < 1 \text{s} \text{ and } \gamma T_\alpha^2 < 0.4 \text{s} \forall \alpha \]
• Renewable energy sources correlation time > few seconds [1]
  \[ \tau_0 \gtrsim 5 - 10 \text{s} \]

Voltage angle cumulants in the long correlation time limit

Using Eq.(4) with Eqs.(5) and (6),

\[ \lim_{\tau_0 \to \infty} \langle \delta \theta^p \rangle_c = a_p \left( \frac{\sigma}{\lambda_0} \sum_{\alpha \geq 2} u_{\alpha,0} u_{\alpha,1} \right)^p \]

• Independent of inertia and damping parameters.
• Non-Gaussian fluctuations propagate the same as Gaussian ones.

Conclusions

✓ Inertia not that important to mitigate non-Gaussian fluctuations.
✓ Non-Gaussian fluctuations originating from a single source spread in the whole grid.
✓ (Don’t worry, in case of multiple sources, non-Gaussian fluctuations disappear (cf. Ref.[2]).

References: