Propagation of non-Gaussian voltage angle fluctuations in high-voltage power grids

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High-voltage power grids of the future

- **More fluctuations from renewable energy sources, possibly non-Gaussian.**
- **Less inertia at generator nodes.**

Goals

➢ **Predict the propagation of fluctuations coming from renewable energy sources.**
➢ **Clarify the role of damping and inertia.**

In the following, we investigate voltage angle fluctuations in realistic high-voltage power grids, induced by renewable energy sources.

Dynamics of AC power grids

Linearized Swing Equations

$$\boldsymbol{M}\delta\ddot{\boldsymbol{\theta}} = -\boldsymbol{D}\delta\dot{\boldsymbol{\theta}} + \delta\boldsymbol{p}(t) - \boldsymbol{L}\delta\boldsymbol{\theta}, \tag{1}$$

with $\boldsymbol{\theta}(t) = \boldsymbol{\theta}_i^{(0)} + \delta\boldsymbol{\theta}(t)$, $\boldsymbol{M} = \mathrm{diag}(\{m_i\})$ and $\boldsymbol{D} = \mathrm{diag}(\{d_i\})$, and the weigthed Laplacian matrix

$$\mathbf{L}_{ij} = \begin{cases} -B_{ij}\cos(\theta_i^{(0)} - \theta_j^{(0)})\,, & \text{for } i \neq j\,, \\ \sum_k B_{ik}\cos(\theta_i^{(0)} - \theta_j^{(0)})\,, & \text{for } i = j\,. \end{cases} \tag{2}$$

Modal decomposition over the eigenmodes of L

$$m\,\ddot{c}_\alpha + d\,\dot{c}_\alpha + \lambda_\alpha c_\alpha = \delta\boldsymbol{p}(t)\cdot\mathbf{u}_\alpha\,, \tag{3}$$

$$\begin{aligned} c_\alpha(t) = {} & m^{-1}e^{-(\gamma+\Gamma_\alpha)t/2}\int_0^t e^{\Gamma_\alpha t_2} \\ & \times \int_0^{t_2} e^{(\gamma-\Gamma_\alpha)t_1/2}\,\delta\boldsymbol{p}(t_1)\cdot\mathbf{u}_\alpha\,\mathrm{d}t_1\mathrm{d}t_2\,, \end{aligned} \tag{4}$$

with $\Gamma_\alpha = \sqrt{\gamma^2 - 4\lambda_\alpha/m}$ and $\gamma = d/m$.

Modelling fluctuations of renewable energy sources

Cumulants of time-correlated noise

$$\langle \delta p_i(t_1)\rangle = 0\,, \tag{5a}$$

$$\langle \delta p_i(t_1)\delta p_j(t_2)\rangle = \sigma^2\,\delta_{ij}\delta_{ii_0}e^{-|t_1-t_2|/\tau_0}\,. \tag{5b}$$

$$\langle \delta p_i(t_1)\delta p_j(t_2)\delta p_k(t_3)\rangle = a_3\,\sigma^3\,\Delta_{ijk}\delta_{ii_0}\,, \tag{6a}$$

$$\langle \delta p_i(t_1)\delta p_j(t_2)\delta p_k(t_3)\delta p_l(t_4)\rangle_c = a_4\,\sigma^4\,\Delta_{ijkl}\delta_{ii_0}\,, \tag{6b}$$

Time-scales

- **Intrinsic network time-scales < few seconds**

$$\gamma^{-1} \simeq 2.5s,\; T_\alpha < 1s \text{ and } \gamma T_\alpha^2 < 0.4s\ \forall\alpha$$

- **Renewable energy sources correlation time > few seconds [1]**

$$\tau_0 \gtrsim 5 - 10s$$

Voltage angle cumulants in the long correlation time limit

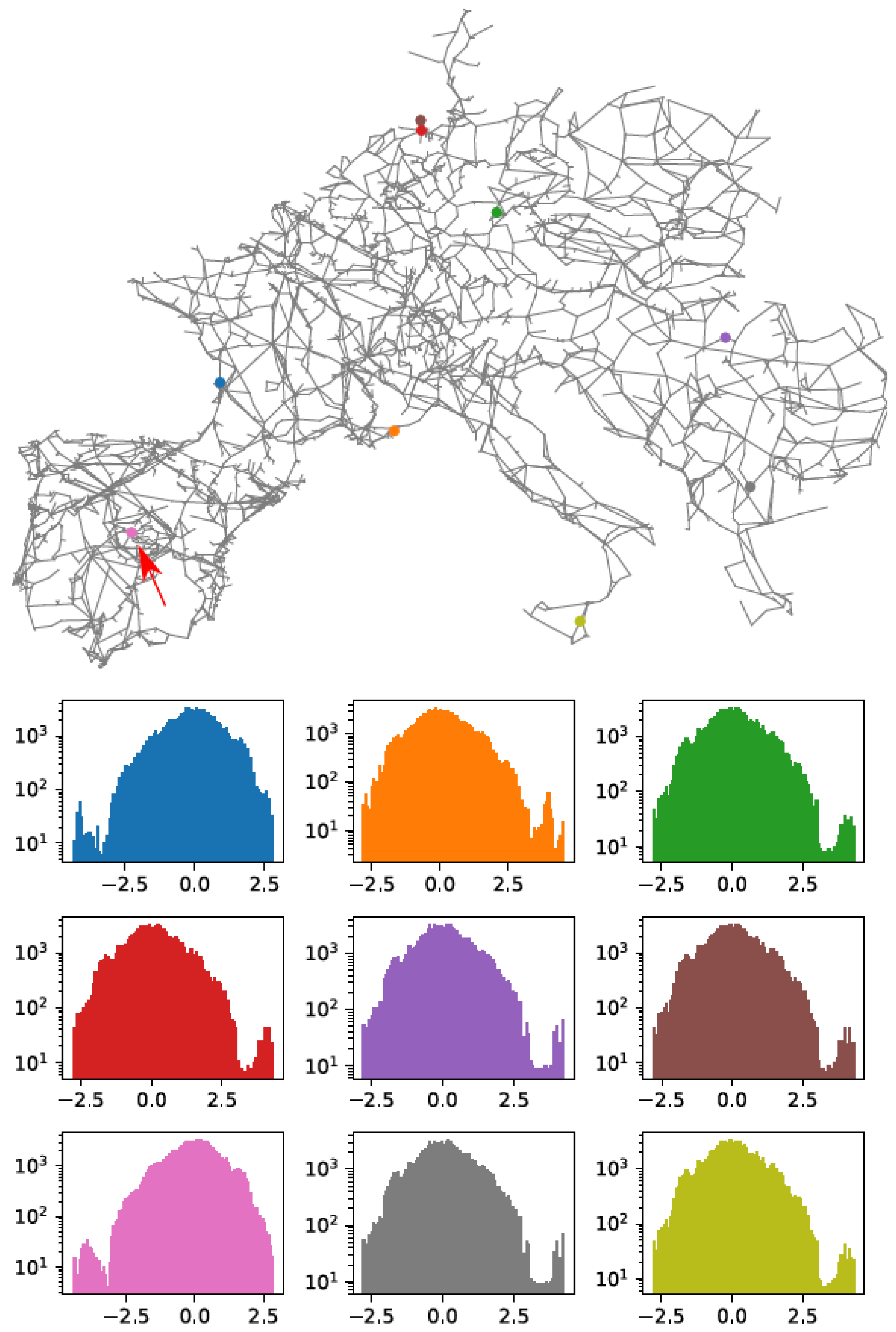
Using Eq.(4) with Eqs.(5) and (6),

$$\lim_{\tau_0\to\infty}\langle \delta\theta_i^p\rangle_c = a_p\left(\sigma\sum_{\alpha\geq 2}\frac{u_{\alpha,i_0}u_{\alpha,i}}{\lambda_\alpha}\right)^p$$

- Independent of inertia and damping parameters.
- Non-Gaussian fluctuations propagate the same as Gaussian ones.

Non-Gaussian fluctuations from a single renewable energy source

Voltage angle distributions.



Conclusions

✓ **Inertia not that important to mitigate non-Gaussian fluctuations.**
✓ **Non-Gaussian fluctuations originating from a single source spread in the whole grid.**
✓ **(Don't worry, in case of multiple sources, non-Gaussian fluctuations disappear (cf. Ref.[2]).**

References:
1. M. Tyloo, P. Jacquod IEEE Control Systems Letters 5 (3), 929-934 (2020)
2. M. Tyloo, J. Hindes, P. Jacquod arXiv preprint arXiv:2203.00590 (2022)